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RATIONAL STEEL CANOPY STRUCTURES OVER STADIUM STANDS USING WELDED I-BEAMS WITH VARIABLE FLANGE WIDTH AND WEB HEIGHT

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Abstract. An improved methodological approach has been developed for determining the optimal structural form of a cantilever welded steel I-beam with variable flange width and web height under combined loading: uniformly distributed along the length and an applied concentrated bending moment at the free end. These steel structures demonstrate a high efficiency of steel use, serving as primary load-bearing elements of canopies over stadium stands.

The structural efficiency is achieved through redistribution of steel along the length of the element and over the section height in such a way as to attain an optimal structural form. The selection of the optimal configuration is based on numerical studies focused on determining the rational gradient of the I-beam's height variation and selecting the gradient value of flange width, while satisfying the strength conditions for each cross-section. It is assumed that the rational distribution of steel over the section height corresponds to the optimal ratio between the flange and web areas according to the criterion of minimizing steel consumption for each elementary segment of the beam.

The optimization problem of a conical welded I-beam under the adopted loading conditions with a linear variation of flange width and web height is formulated in the traditional manner as a single-criterion constrained optimization problem, where the criterion is the minimization of steel consumption, and strength constraints in each cross-section (which depend on the gradients of variation of flange width and web height under combined loading) are imposed.

The problem is solved with the aim of complying with the strength constraints and determining the limiting values of the gradients of change of web height and flange width of the cross section of the steel I-beam for a generalized loading scheme.

Numerical investigations were carried out to find the gradients of variation of web height and flange width based on the criterion of minimizing steel consumption and satisfying strength constraints in each cross-section along the length of the structure.

The results obtained show that the optimal gradient of variation of section height is $\gamma_h \leq 0{,}6$ for $P_1 l/M_{x,0} \leq 2{,}0$, and $\gamma_h \leq 0{,}5$ for $P_1 l/M_{x,0} \leq 1{,}0$.. It has been established that for $P_1 l/M_{x,0} \leq 0{,}25$ and $\gamma_h \leq 0{,}9$, the strength conditions are satisfied for all cross-sections of the rational structural form of the welded steel I-beam. A methodological approach has been developed for solving problems of this type in a generalized form.

Keywords: steel structures; stadium coverings; optimal web height and flange width of a welded I-beam and their variation gradient; modeling; numerical studies.

PROBLEM STATEMENT

Spatial cantilever steel structural systems are predominantly used for canopies over stadium stands.

This is due to the functional purpose of canopies: protecting spectators from atmospheric precipitation and providing shade for the stadium seating areas. The cantilever span of such structures always exceeds 8–10 m, and the extensive use of these systems highlights the relevance of the task of identifying the most steel-efficient main load-bearing beam structure. The research presented in this paper continues the author's previous works published in [5, 10, 11]. The scientific novelty lies in the generalization of the problem formulation in terms of analyzing and identifying the most rational structural form under the action of combined loading: uniformly distributed load and concentrated loads applied at the free end.

ANALYSIS OF PREVIOUS RESEARCH

In previous research, the author performed an analysis of methodological approaches for determining the best structural form of a welded steel I-beam with variable web height and flange width under uniformly distributed loading [1–3, 9–11, 14, 16, 18, 22–24]. It was shown in [5, 10, 11] that most often the criterion of minimum material consumption (by a reduced parameter) [5, 10, 11, 12, 22] and the criterion of safe long-term performance under static and seismic loads [19] are used. For critical structures, the criterion of minimum steel consumption is combined with the natural vibration frequencies and the shape of the variable cross-section [6, 13, 23]. The criterion of uniform stress (constant stress) in the most heavily loaded sections is used for investigating combined steel systems [12, 15]. Works [10, 11, 14, 16, 18, 21, 23, 24], including those with corrugated web [20] and flexible web sections, are devoted to finding the optimal height of constant cross-section beams. This article is a continuation of research on such structures previously conducted by the author at the Department of Metal and Timber Structures of KNUBA [5, 10, 11].

PURPOSE AND METHODS

The purpose of the research is to determine the optimal structural form of a conical steel welded I-beam with linear variation of flange width and web height according to the criterion of minimal steel consumption, while satisfying the strength conditions at each cross-section along the length of the structure under combined loading: a uniformly distributed load and a concentrated vertical transverse force at the free end.

To achieve the stated objective, the methodology of Lagrange multipliers is used to find the optimum of the objective function in terms of steel consumption, along with the necessary Kuhn-Tucker conditions for the optimal solution. The flexibility of the beam's web is variable along its length; the flexibility of the web at the support cross-section is .

$$\lambda_\omega \approx h_0/t_\omega.$$

Local buckling (stability) of the flanges and web is assumed to be ensured. The dimensions of the support cross-section, where the maximum bending moment occurs, are denoted as (b_{f0}), (h_0).

The bending moment variation is assumed according to equation (1).

$$M_{x,z} = M_{x,0}(1-\frac{z}{l})^2 + P_1 l(1-\frac{z}{l}). \qquad (1)$$

The flange width (b_{fz}) and web height (h_z) of the variable cross-section steel beam vary linearly along the length of the structure from the support section to the free end:

$$b_{f,z} = b_{f,0}\left(1-\frac{\gamma_b\, z}{l}\right); h_z = h_0\left(1-\frac{\gamma_h\, z}{l}\right);$$

l– is the span of the steel beam.

The web thickness is assumed constant (2). The web slenderness of the beam is variable along the length; the slenderness of the web at the support section is given by an expression in (2) $\lambda_\omega \approx h_0 / t_\omega$.

$$h_z = h_0 f_{hz} = h_0\left(1-\gamma_h\frac{z}{l}\right); b_{f,z} = b_{f,0} f_{bz} = b_{f,0}\left(1-\frac{\gamma_b\, z}{l}\right)$$
$$f_{hz} = (1-\gamma_h\frac{z}{l}). f_{bz} = (1-\gamma_b\frac{z}{l}). \qquad (2)$$

The spatial stability of the beam against loss of stability in the plane of bending is ensured by out-of-plane bracing.

The optimization problem of the conical welded I-beam with linear variation in flange width and web height is formulated as a single-criterion constrained optimization problem, where the objective is minimization of steel consumption [10]. To identify a saddle (stationary) point, strength constraints in each cross-section of the structure are imposed. Since satisfying these strength conditions requires finding two independent parameters – the rational distribution of steel along the cross-section depth and the gradient of cross-section variation for the flange and web (γ_h , γ_b) according to (2) – the problem is converted to an unconstrained optimization. Following the findings in [10] he assumption is made that if the beam is optimal in each cross-section, then the entire structure is optimal as well. However, with linear variation of flange width and web height, the strength constraints take the form of an equality for the cross-section where the maximum stress occurs, and inequalities for the other sections. «The problem is tackled in two stages: first, using Lagrange multipliers, it was determined [10], that under linear variation of flange width and web height, the optimal cross-sectional geometry satisfies: $2A_{f,z} = 2b_{f,z}t_f = h_zt_w; 2b_{f,0}t_f = h_0t_w$ - the area of the two flanges equals the area of the web, and the variation gradients of flange width and web height are equal ($\gamma_h = \gamma_b$)». In the second stage, numerical studies determined the limiting values of the cross-section variation gradient for the steel I-beam.

The stresses in each cross-section of the steel I-beam structure are given by

$$\sigma_z = \frac{\left(M_{x,z} + P_{1z}\right)}{h_0^2 t_{w,0}(1-\gamma_h \frac{z}{l})^2 \left(\frac{t_f b_{f0}(1-\gamma_b \frac{z}{l})}{t_{w,0}h_0(1-\gamma_h \frac{z}{l})} + \frac{1}{6}\right)} \le R_y\gamma_c. \tag{3}$$

Taking into account the equality of the web height and flange width variation gradients ($\gamma_h = \gamma_b$) the strength condition can be written in the form:

$$\sigma_z = \frac{M_{x,0}}{h_0^2 t_{w,0}} \frac{\left[(1-\frac{z}{l})^2 + \frac{P_1 l}{M_{x,0}}(1-\frac{z}{l})\right]}{(1-\gamma_h \frac{z}{l})^2 \left(\frac{t_f b_{f0}(1-\gamma_h \frac{z}{l})}{t_{w,0}h_0(1-\gamma_h \frac{z}{l})} + \frac{1}{6}\right)} \le R_y\gamma_c. \tag{4}$$

Applying the optimality conditions yields $2A_{f,z} = 2b_{f,z}t_f = h_zt_w; 2b_{f,0}t_f = h_0t_w$ the analytical expression:

$$\frac{2t_f b_{f0}(1-\gamma_h \frac{z}{l})}{t_{w,0}h_0(1-\gamma_h \frac{z}{l})} = 1 \to \frac{M_{x,z} + P_{1z}}{t_{w,0}h_0(1-\gamma_h \frac{z}{l})^2 \left(\frac{1}{2} + \frac{1}{6}\right)} = R_y\gamma_c \tag{5}$$

$$\frac{3}{2}\frac{M_{x,z} + P_{1z}}{t_{w,0}h_0(1-\gamma_h \frac{z}{l})^2} = R_y\gamma_c. \tag{6}$$

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Equation (6), the last strength condition, gives a formula for determining the optimal height of the support section, as for steel I-beams of constant cross-section. [15]

Since, depending on the variation gradient of the cross-section, the strength condition may not be satisfied in some sections, analytical and numerical studies were conducted using the criterion that the stresses do not exceed the yield strength of the steel. In other words, the strength constraints serve as the criterion for determining the cross-section variation gradient.

$$z = 0 \rightarrow h_0 = \sqrt{\frac{3}{2}\frac{M_{x,z} + P_z}{t_{w,0} R_y \gamma_c}}.$$

$$\frac{\sigma_z}{R_y \gamma_c} = \frac{\left[(1-\frac{z}{l})^2 + \frac{Pl}{M_{x,0}}(1-\frac{z}{l})\right]}{\left[1+\frac{Pl}{M_{x,0}}\right](1-\gamma_h \frac{z}{l})^2} \leq 1. \tag{7}$$

Now the gradient of variation of the web height and flange width must satisfy an algebraic inequality:

$$\gamma_h \leq 1 - \frac{l}{z}\sqrt{\frac{\left[(1-\frac{z}{l})^2 + \frac{Pl}{M_{x,0}}(1-\frac{z}{l})\right]}{\left[1+\frac{Pl}{M_{x,0}}\right]}}. \tag{8}$$

Numerical studies using analytical expressions (7) and (8) were carried out to determine the maximum value of the variation gradient of the web height and flange width for the conical beam.

He calculation results showed (Fig. 1) that the optimal gradient of cross-section change at $Pl/M_{x,0}$=1,0 should be considered γ_h=0,7. With this topology, the strength conditions are satisfied in all cross-sections along the length of the cantilever conical I-beam structure.

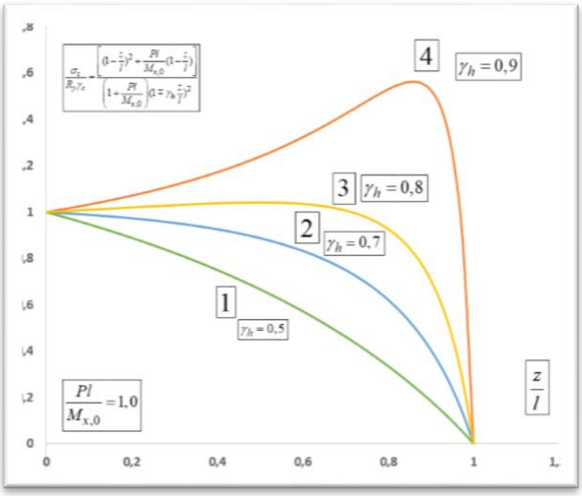


Fig. 1 Ratio of current stress to the design steel strength in a welded steel I-beam with symmetric cross-section at different gradients of linear variation of web height and flange width (2.52) and at a ratio of bending moments from external loads $Pl/M_{x,0}$=1,0: graph 1 - $\gamma_h \leq 0{,}5$; graph 2 - γ_h=0,7; graph 3 - γ_h=0,8; graph 4 - γ_h=0,9.

Рис. 1 Відношення поточних напружень до розрахункового опору сталі у зварному сталевому двотаврів симетричного перерізу при різних градієнтах лінійній змінності висоти стінки і та полиці (2.52) та відношенні між згинальними моментами від зовнішніх зусиль $Pl/M_{x,0}$=1,0: графік 1 - $\gamma_h \leq 0{,}5$; графік 2 - γ_h=0,7; графік 3 - γ_h=0,8; графік 4 - γ_h=0,9.

With an increase in the ratio of bending moments from external loads to 2.0, i.e. $Pl/M_{x,0}$=2,0 (Fig. 2), the optimal value of the cross-section variation gradient is γ_h=0,6. Thus, it is established that with an increasing influence of the linear distribution of bending moments from concentrated forces, the flange width and web height variation gradients decrease, and the optimal I-beam topology becomes flatter.

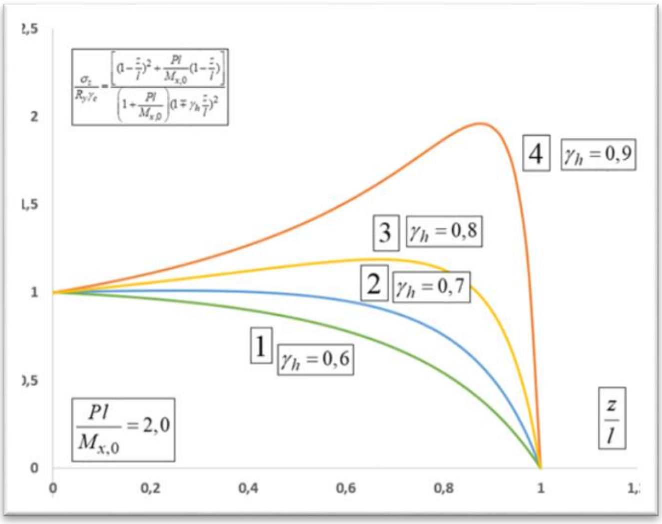
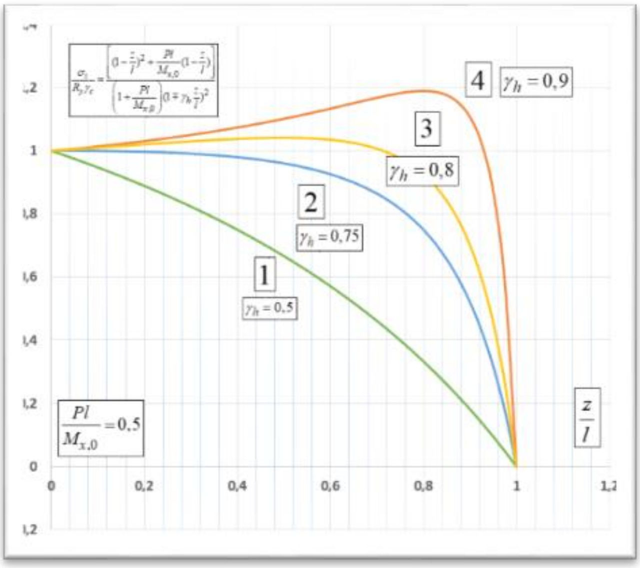


Fig. 2 Ratio of current stress to the design steel strength in a welded steel I-beam with symmetric cross-section at different gradients of linear variation of web height and flange width (2.52) and at a ratio of bending moments from external loads $Pl/M_{x,0}$=2,0: graph 1 - γ_h≤0,5; graph 2 - γ_h=0,7; graph 3 - γ_h=0,8; graph 4 - γ_h=0,9.

Рис. 2 Відношення поточних напружень до розрахункового опору сталі у зварному сталевому двотаврів симетричного перерізу при різних градієнтах лінійній змінності висоти стінки і та полиці (2.52) та відношенні між згинальними моментами від зовнішніх зусиль $Pl/M_{x,0}$=2,0: графік 1 - γ_h≤0,5; графік 2 - γ_h=0,7; графік 3 - γ_h=0,8; графік 4 - γ_h=0,9.

Additional studies were carried out at lower values of the ratio $Pl/M_{x,0}$= 0.5 (Fig. 3) and $Pl/M_{x,0}$= 0.25 (Fig. 4). The results show that when the applied load ratio decreases to $Pl/M_{x,0}$= 0.5 (Fig. 3), a more conical structural form with γ_h = 0.75 can be considered rational.



As the ratio approaches $Pl/M_{x,0}$= 0.25 (Fig. 4), a sufficiently rapid transition to the optimal conical beam form occurs: at γ_h = 0.9, the strength conditions are satisfied along the entire length of the structure.

Fig. 3. Ratio of actual stresses to the design steel strength in a welded steel I-beam with a symmetrical cross-section under various gradients of linear variation in web height and flange width (2.52), and the ratio of bending moments from external loads $Pl/M_{x,0}$=0,5: curve 1 - γ_h≤0,5; curve 2 - γ_h=0,75; curve 3 - γ_h=0,8; curve 4 - γ_h=0,9

Рис. 3 Відношення поточних напружень до розрахункового опору сталі у зварному сталевому двотаврів симетричного перерізу при різних градієнтах лінійній змінності висоти стінки і та полиці (2.52) та відношенні між згинальними моментами від зовнішніх зусиль $Pl/M_{x,0}$=0,5: графік 1 - γ_h≤0,5; графік 2 - γ_h=0,75; графік 3 - γ_h=0,8; графік 4 - γ_h=0,9.

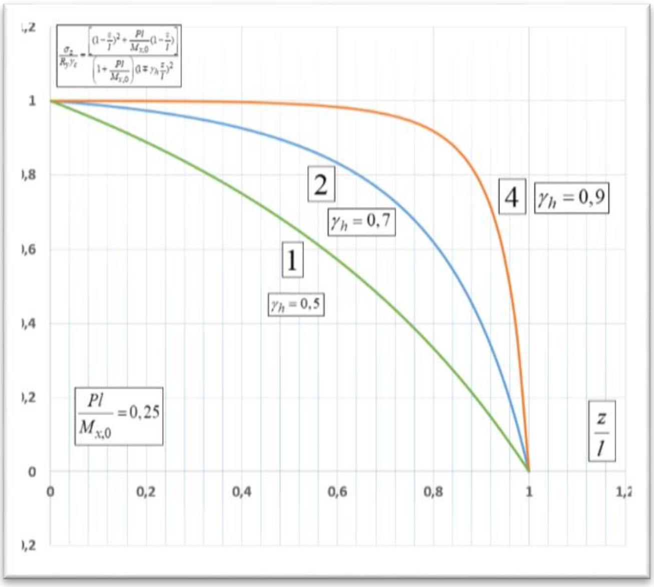


Fig. 4. Ratio of actual stresses to the design steel strength in a welded steel I-beam with a symmetrical cross-section under various gradients of linear variation in web height and flange width (2.52), and the ratio of bending moments from external loads $Pl/M_{x,0}$= 0.5: curve 1 - $\gamma_h \le 0.5$; curve 2 - $\gamma_h = 0.7$; curve 3 - $\gamma_h = 0.9$

Рис. 4 Відношення поточних напружень до розрахункового опору сталі у зварному сталевому двотаврів симетричного перерізу при різних градієнтах лінійній змінності висоти стінки і та полиці (2.52) та відношенні між згинальними моментами від зовнішніх зусиль $Pl/M_{x,0}$=0,5: графік 1 - $\gamma_h \le 0,5$; графік 2 - γ_h=0,7; графік 3 - γ_h=0,9.

CONCLUSIONS AND RECOMMENDATIONS

The results obtained indicate that the rational gradient of cross-section variation is γ_h=0,6 for $P_1l/M_{x,0} \le 2,0$ and γ_h=0,5 for $P_1l/M_{x,0} \le 1,0$. A methodological approach for finding the optimal design of a welded I-beam with variable cross-section under linear variation of flange width and web height has been developed.

A more conical structural form of the I-beam with γ_h = 0.75 can be considered a rational design solution. As the ratio approaches $Pl/M_{x,0}$= 0.25 (Fig. 4), a sufficiently rapid transition to the optimal conical form of the steel I-beam occurs: at γ_h = 0.9, the strength conditions are satisfied along the entire length of the structure.

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РАЦІОНАЛЬНІ СТАЛЕВІ КОНСТРУКЦІЇ НАВІСІВ НАД ТРИБУНАМИ СТАДІОНІВ ІЗ ВИКОРИСТАННЯМ ЗВАРНИХ ДВОТАВРІВ ІЗ ЗМІННОЮ ШИРИНОЮ ПОЛИЦІ І ВИСОТИ СТІНКИ

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Анотація. Розроблено удосконалений методичний підхід для визначення оптимальної конструктивної форми консольної зварної сталевої двотаврової балки зі змінною шириною полиці та висотою стінки при комбінованому навантаженні: рівномірно розподіленому по довжині та зосередженому згинальному моменті, прикладеному на вільному кінці. Такі сталеві конструкції демонструють високу ефективність використання сталі, виступаючи основними несучими елементами покриттів над трибунами стадіонів.

Ефективність конструкції досягається завдяки перерозподілу сталі по довжині елемента та по висоті поперечного перерізу з метою досягнення оптимальної конструктивної форми. Підбір оптимальної конфігурації базується на числових дослідженнях визначення раціонального градієнта змінності висоти балки та підбору значення градієнта ширини полиці з урахуванням виконання умов міцності для кожного перерізу. Вважається, що раціональний розподіл сталі по висоті перерізу відповідає оптимальному співвідношенню площі полиці і стінки за критерієм мінімізації витрат сталі на кожну елементарну ділянку балки.

Задача оптимізації зварного двотавра конічної форми при прийнятих умовах навантаження з лінійною зміною ширини полиці та висоти стінки сформульована традиційно як однокритеріальна задача умовної оптимізації, в якій критерієм є мінімізація витрат сталі за умов дотримання умов міцності у кожному поперечному перерізі (що залежать від градієнтів змінності ширини полиці та висоти стінки при комплексному навантаженні).

Задача вирішується з метою дотримання обмежень умов міцності та визначення граничних значень градієнта зміни висоти стінки та ширини полиці в поперечному перерізі сталевої двотаврової балки для узагальненої схеми навантаження. Проведено числові дослідження для визначення градієнтів змінності висоти стінки та ширини полиці за критерієм мінімізації витрат сталі та дотримання умов міцності в кожному перерізі вздовж довжини конструкції.

Отримані результати показують, що оптимальним градієнтом змінності висоти перерізу є $\gamma_h \leq 0{,}6$ при $P_1 l/M_{x,0} \leq 2{,}0$ та $\gamma_h \leq 0{,}5$ при $P_1 l/M_{x,0} \leq 1{,}0$. Встановлено, що при $P_1 l/M_{x,0} \leq 0{,}25$ та $\gamma_h \leq 0{,}9$ умови міцності виконуються для всіх перерізів раціональної конструктивної форми зварної сталевої балки. Розроблено методологічний підхід до вирішення такого типу задач в узагальненому вигляді.

Ключові слова: сталеві конструкції; покриття стадіонів; оптимальна висота стінки та ширина полиці зварного двотавра і градієнт їх змінності; моделювання; числові дослідження

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