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HISTORY OF EMERGENCE AND DEVELOPMENT OF THE SANDWICH MODEL FOR DESIGN OF MEMBRANE, SHELL AND SLAB ELEMENTS ACCORDING TO EN 1992-1-1:2023

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Abstract. For the first time in European design standards for concrete structures, specifically in Annex G (normative), the new version of Eurocode EN 1992-1-1:2023 pays focused attention to the design of membrane, shell and slab elements and proposes a modern approach for their calculation.

As a method of optimal design for reinforced concrete shells, slabs and membranes, Annex G proposes the so-called **sandwich model** - where the shell is represented as a three-layer model consisting of two load-bearing layers (top and bottom) and an intermediate layer between them.

When designing using the sandwich model – a spatial problem (bending + torsion + membrane forces) is transformed into two membrane problems for the top and bottom layers. That is, transformation of a three-dimensional stress state into two layers (top and bottom) with equivalent membrane stresses.

The sandwich model is a rigorous mechanical model that allows reducing a complex spatial problem (shell with combined forces) to two independent plane problems (membrane elements in the top and bottom layers) through statically equivalent transformation of forces into stresses.

The sandwich model is based on transforming the combination of forces (membrane, bending and torsional) into a statically equivalent system of in-plane stresses acting in the top and bottom layers of the model.

Basic assumptions of the method:

a) three-layer model: the shell is represented as a structure with two load-bearing layers (top and bottom) and an intermediate layer between them;

b) static equivalence: internal forces are transformed into in-plane stresses in such a way as to maintain complete static equilibrium;

c) independent layer design: each layer (top and bottom) is designed separately as a membrane element according to clause G.3.

The formulations presented in Annex G of EN 1992-1-1:2023 in clauses G.3 and G.4 are consistent with the clauses and design provisions in Section 8 (Ultimate Limit States (ULS)) of the main body of the document, and clause G.5 contains additional provisions to 9.2 (Crack Control) of these standards. Annex G covers the design of planar members without discontinuities in the concrete mass. Other more refined design methods complying with clause 7.3.3 (Plastic Analysis) or clause 7.3.4 (Non-linear Analysis) of Eurocode 2 may be used.

Thus, Annex G (Design of Membrane, Shell and Slab Elements) does not replace, but supplements the main sections of Eurocode 2, namely - general principles, materials (σ-ε diagrams), cross-section design for ULS, crack resistance and SLS.

Keywords: design of membrane; shell and slab elements; sandwich model..

PROBLEM STATEMENT

For the first time, the so-called sandwich model for optimal design of reinforced concrete shells and slabs was proposed in 1974 by Danish scientist Troels Brondum-Nielsen [1]

In his proposed design method, an infinitesimal shell element was divided by height into three layers, including two outer layers and a core layer, called the three-layer **Basic Sandwich Model (BSM)**. In this approach, membrane forces, bending and torsional moments are carried only by the outer layers of the shell.

The Basic Sandwich Model (BSM) has disadvantages due to neglecting different lever arms of all forces and the assumption that the resultant forces in steel act in the mid-plane of the outer layer, which can lead to unsafe design at large torsional moments or high reinforcement ratios.

The core in the basic sandwich model is considered to carry transverse shear forces. The design implementation assumes the absence of diagonal cracks in the core. Under this assumption, a state of pure shear develops in the core, which means that the transverse forces in the section do not affect the membrane forces in the outer layers.

Subsequently, Swiss scientist A. Peter Marti made his contribution to the development of the sandwich model in 1990 [2]. Marti complements Brondum-Nielsen's work by assigning out-of-plane shear forces to the intermediate layer. The three-layer sandwich model was then developed for the design of members subjected to membrane and bending states and to account for transverse shear [3]. The outer layers of the sandwich model (i.e., the outer layers) are considered to carry moments and membrane forces, while transverse shear forces are assigned to the core [4].

The key innovation of Marti's contribution to the development of the sandwich model was the distribution of functions between layers:

- **outer layers** – carry membrane forces + bending moments;
- **core** – carries transverse forces through compression fields inclined at 45°.

The inclination angle of the diagonal compression stress field in the core layer θ is taken equal to 45°, which corresponds to the traditional Mörsch truss model for reinforced concrete beams.

This leads to additional membrane forces in the top and bottom layers of the slab, equal to $0.5v_0$ in the direction of the principal transverse force.

Later in 2013-2014, Swiss researcher Thomas Jaeger presented the "**Extended Sandwich Model (ESM)**", which combines the sandwich model with basic concepts of the cracked membrane model and a new aggregate interlock relationship for shear stresses transmitted through cracks in the core [5-7].

Thomas Jaeger introduces a compatibility condition to link both outer layers, assuming a linear distribution of strains through the slab thickness, defined as a function of three curvatures and three strains in the mid-plane of the core.

The Extended Sandwich Model includes aggregate interlock in the core, allowing concrete to contribute to the transverse load-bearing capacity of slabs both with and without transverse reinforcement.

The introduction of stressed crack faces in the core allows dividing the applied transverse force into concrete and steel contributions.

Thomas Jaeger's contribution to the development of the sandwich model consisted of the following:

- accounting for cracking in the core;
- aggregate interlock;
- compatibility conditions between layers;
- tension stiffening in the outer layers.

In 2014-2018, E. Hernández-Montes and colleagues conducted a critical analysis and questioned the assumption of reinforcement yielding in the outer layers, which was adopted by Brondum-Nielsen and Marti.

The sandwich model of E. Hernández-Montes concerns an improved model for analysis of reinforced concrete slabs that accounts for limitations of the basic sandwich model. The model accounts for compatibility between tensile reinforcement and the compressive concrete block, which is crucial for accurate strength calculation, especially in situations with high torsional moments [10-15].

The classical sandwich model assumes that tensile reinforcement reaches yielding ($\sigma_s = f_y$), but this hypothesis was challenged by E. Hernández-Montes and others, who proposed that the strain in tensile reinforcement should be related to the depth of the compressive stress block in the opposite layer.

Physical meaning: If concrete in the top layer fails before reinforcement in the bottom layer reaches yielding, the structure will have less load-bearing capacity and less deformability than calculated.

E. Hernández-Montes proposed an addition-nal check to ensure that reinforcement actually reached yielding.

Key aspects of the Hernández-Montes model:

- **strain compatibility:** the model refutes the assumption that tensile reinforcement in reinforced concrete slabs always undergoes yielding, proposing instead that its strain is related to the depth of the concrete compressive stress block. This provides a more accurate stress value for reinforcement.
- **stress verification:** it introduces an additional verification step based on the plane section hypothesis. It assumes that the principal compression direction in one outer layer is parallel to the principal tension direction in the opposite outer layer.
- **upper bound theorem:** the model's assumptions ensure that calculated forces are an upper bound of true failure forces, which is a safe design approach.
- **experimental verification:** the accuracy of this approach was experimentally verified by Gil-Martín and Hernández-Montes.
- **basis for advanced models:** the concepts of this work led to the development of the **Advanced Sandwich Model (ASM)**, which solves many shortcomings of the basic sandwich model, such as those related to torsional moments and reinforcement ratios.
- **applications:** this research is particularly relevant for calculating the strength of reinforced concrete slabs subjected to bending and torsional moments. Its principles have also been applied in other areas, such as seismic behavior of coupled walls subjected to shear, and thin reinforced concrete panels [8-12].

The main contribution to the development of the sandwich model by E. Hernández-Montes and colleagues:

- verification of **strain compatibility** between layers;
- **introduction of limitations** on the method's application.

In 2023, a group of researchers [9] proposed the so-called **Developed Advanced Sandwich Model (DASM)**.

This study showed that using DASM reduced steel reinforcement by up to 40% and increased ductility by 10-15%. It was also observed that the ultimate strength of the considered examples, including solid slab and flat slab, decreased slightly by 4.1% and 1.8% (less than 5%) respectively, which has almost no effect on the overall response of the designed structure. These results demonstrate the effectiveness of the developed procedure.

DASM represents a highly accurate, relatively simple and reliable design procedure for plate and shell structures with complex geometry according to multiaxial concrete compression state and accounting for the effect of transverse shear forces.

The first generation of Eurocodes EN 1992-1-1:2004 (2002-2007) did not contain the sandwich method in normative annexes. Although this method was known, it was not standardized.

The second generation of Eurocodes EN 1992-1-1:2023 (2023-2026) contains **Annex G (normative)** – «Design of Membrane, Shell and Slab Elements». This document (EN 1992-1-1:2023) was prepared by Technical Committee CEN/TC 250 «Structural Eurocodes».

In Annex G of EN 1992-1-1:2023, a planar member, depending on the internal forces acting in it, is considered:

- as a **membrane** – a slab with normal and shear stresses (clause G.3) – see Fig. 1;
- as a **shell** – a slab with moments without membrane forces (clause G.4) – see Fig. 2;

- **as a shell or slab with combined forces** (clauses G.4, G.5) – see Fig. 2.

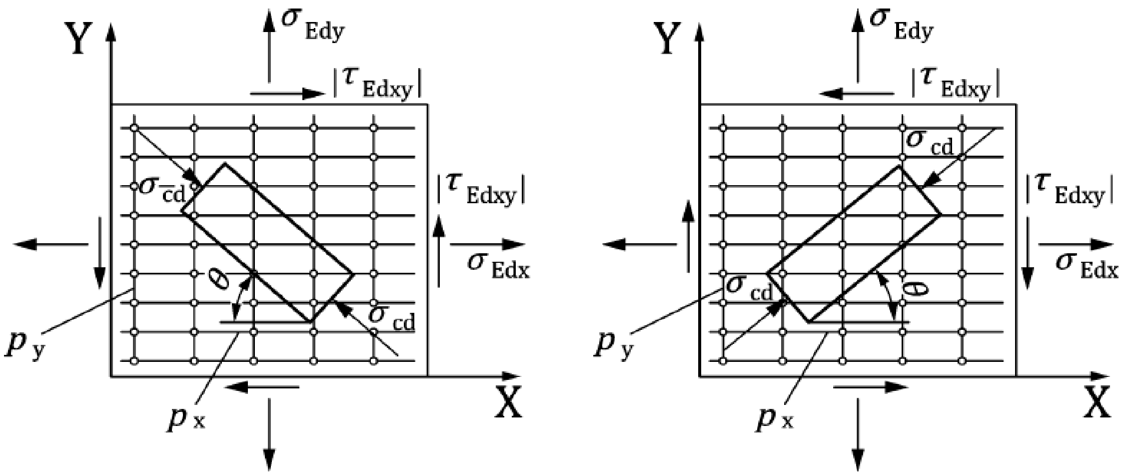
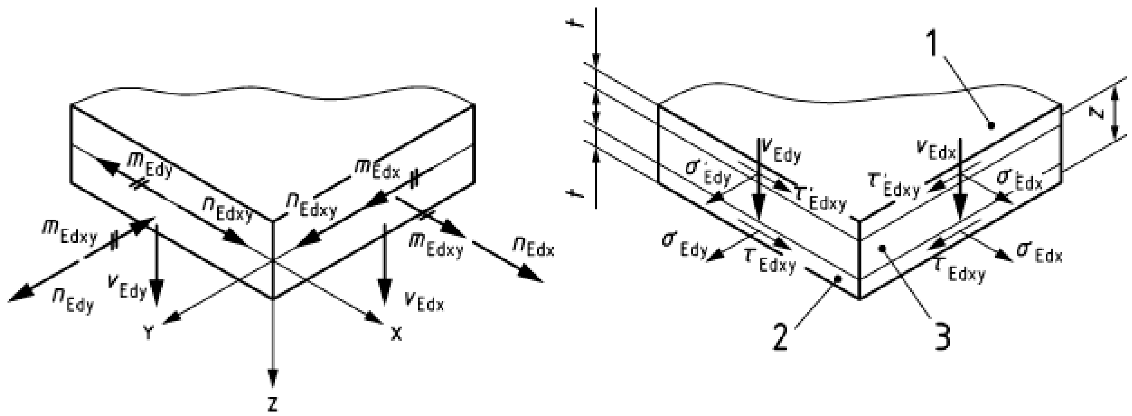


Fig. 1 In-plane stresses in membrane element and definition of compression stress field inclination (angle θ).

Рис. 1 Площинні напруження в мембранному елементі та визначення нахилу (кут θ) поля стискаючих напружень.



a б

Fig. 2 Shell element (a) and sandwich model (b) with statically equivalent set of in-plane stresses: 1 – top layer, 2 – bottom layer, 3 – intermediate layer.

Рис. 2 Оболонковий елемент (а) та сендвіч-модель (б) зі статично еквівалентним набором площинних напружень: 1 – верхній шар, 2 – нижній шар, 3 – проміжний шар.

The formulations presented in Annex G of EN 1992-1-1:2023 in clauses G.3 and G.4 are consistent with the clauses and design provisions in Section 8 (Ultimate Limit States (ULS)) of the main body of the document, and clause G.5 contains additional provisions to 9.2 (Crack Control) of these standards. Annex G covers the design of planar members without discontinuities in the concrete mass. Other more refined design methods complying with clause 7.3.3 (Plastic Analysis) or clause 7.3.4 (Non-linear Analysis) of EN 1992-1-1:2023 may be used. Thus, Annex G (Design of Membrane, Shell and Slab Elements) does not replace, but supplements the main sections of

Eurocode 2 EN 1992-1-1:2023, namely – general principles, materials (σ-ε diagrams), cross-section design for ULS, crack resistance and SLS.

Key differences of the new version of standards from the previous version of Eurocode 2 are as follows:

- refined formulas for stresses in layers;
- more detailed tables of optimum reinforcement;

- explicit condition for application of simplified method: $|m_{Edxy}| < 0{,}07d^2f_{cd}$;
- references to specific clauses of other sections.

EN 1992-1-1:2023 uses various methods and models for stress-strain state analysis of structures:

- linear elastic analysis (LEA);
- non-linear finite element analysis (NLFEA);
- yield line method;
- strip method;
- sandwich model.

In particular, when designing using the strip method -- the slab is considered as a system of orthogonal strips working in one direction. The load is distributed between strips in two directions.

The scope of application of the provisions given in Annex G of EN 1992-1-1:2023 may include flat slabs, slabs of constant or variable thickness, slabs with beams, cylindrical and other shells. This annex is also particularly important for design of:

1. **shear walls** – large shear stresses;
2. **deep beam** – non-linear stress distribution;
3. **slabs with concentrated loads** – zone around columns;
4. **bunkers and tanks** – membrane forces + bending;
5. **shells** – complex three-dimensional stress state.

According to clause G.4(1) of EN 1992-1-1:2023 and Fig. 2, the sandwich model is based on transforming the combination of forces (membrane, bending and torsional) into a statically equivalent system of in-plane stresses acting in the top and bottom layers of the model.

Basic assumptions of the method:

1. **three-layer model**: the shell is represented as a structure with two load-bearing layers (top and bottom) and an intermediate layer between them;
2. **static equivalence**: internal forces are transformed into in-plane stresses in such a way as to maintain complete static equilibrium;
3. **independent layer design**: each layer (top and bottom) is designed separately as a membrane element according to clause G.3.

The sandwich model formulas (G.12-G.17) of EN 1992-1-1:2023 are **transformed** into formulas for specific cases:

1. **pure membrane elements** (clause G.3) – when moments = 0;
2. **slabs without membrane forces** (clause G.4(5), formulas G.18-G.21) – when n = 0;
3. **shells** – general case with all forces.

Thus, the sandwich model is a generalization that covers all three cases.

Physical meaning of the sandwich model:

- **«smears»** the actual stress distribution into two equivalent layers;
- **maintains** equilibrium of forces and moments;
- **allows** using simple membrane formulas (clause G.3).

Advantages:

- unified approach for all types of elements;
- explicit physical interpretation;
- simplicity of calculations;
- possibility of manual calculation.

Disadvantages:

- approximate stress distribution;
- requires adjustment of coefficients (t, z);
- does not account for exact strain distribution.

Why was the sandwich model specifically chosen for implementation in EN 1992-1-1:2023.

1. Widespread use in software.

Many modern design software worldwide use Brondum-Nielsen's approach for calculating reinforcement of concrete shells in the ultimate limit state under bending and membrane forces.

Software using the sandwich method:

- SAP2000;
- ETABS;
- RFEM;
- SCIA Engineer;
- Midas Gen.

2. Need for standardization for shells.

Existing design codes did not provide specific design methods for reinforced concrete slabs where slabs are subjected to both complex forces and moments.

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The problem was that until 2023 there was no standardized method in Eurocode for calculating:

- shells with combination of membrane forces and bending moments;
- bridge slabs with complex load;
- tanks, cooling towers, dome structures.

3. Scientific consensus and experimental database [1-25].

Thomas Jaeger and Peter Marti conducted 28 tests to failure on fourteen reinforced concrete slab specimens to investigate the effect of deviation of principal shear and moment direction from reinforcement direction [4].

Research database:

- dozens of experimental programs (1974 – 2023);
- hundreds of publications in scientific journals;
- method verification on real structures.

4. Simplicity and practicality.

Method advantages:

- statically equivalent;
- does not require complex FEM for ordinary cases;
- can be implemented manually;
- conservative (safe).

5. Integration with other methods of Eurocode EN 1992-1-1:2023**.**

The sandwich method is consistent with:

- clause 8.2 (Shear design);
- clause 9.2 (Crack control);
- clauses 7.3.3 and 7.3.4 (Non-linear analysis) – as an alternative.

Table 1 presents the chronology of sandwich model development.

Table 1. Chronology of sandwich model development
Табл. 1 Хронологія розвитку сендвіч-моделі

Year	**Development**
1974	Brondum-Nielsen: BASIC SANDWICH MODEL (BSM) [Denmark, Technical University]: • three layers • membrane forces + bending moments in outer layers
1990	Peter Marti: DEVELOPMENT FOR TRANSVERSE FORCES [Switzerland, ETH Zürich]: • transverse forces in core • angle θ = 45° for compression fields • publication in ACI Structural Journal.
2004	EN 1992-1-1:2004 (1st generation): • without sandwich method in normative annexes
2010	fib Model Code 2010: • contains concepts of sandwich method • creates scientific consensus
2013-2014	Thomas Jaeger: EXTENDED MODEL (ESM) [Switzerland, ETH Zürich]: • cracking in core • aggregate interlock • strain compatibility
2014-2018	Hernández-Montes: CRITICAL ANALYSIS: • verification of reinforcement yielding • application limitations
2020-2023	Preparation of EN 1992-1-1:2023 (2nd generation) [CEN/TC 250]: • working group on shells and slabs • integration of sandwich method
2023	EN 1992-1-1:2023: OFFICIAL INCLUSION: • Annex G (normative) -- sandwich method • based on Brondum-Nielsen + Marti • simplified version without complex refinements • normative status throughout Europe
2023-2026	Gradual implementation in national standards

<u>What led to the conviction for including the sandwich panel in EN 1992-1-1:2023?</u>

A. Recommendations of fib (International Federation for Structural Concrete).

Fib Model Code 2010 [20] already contained similar concepts, which created scientific consensus.

B. Pressure from practicing engineers.

Engineers were already using the method in software, but there was no official standard. This created:

- legal problems;
- different interpretation;
- lack of uniform acceptance criteria.

C. Successful Swiss experience.

Swiss standards (SIA) already contained Marti's method, which had been successfully used for decades.

D. Mandate M/515 of the European Commission.

Eurocodes were developed under Mandate M/515, issued to CEN by the European Commission and European Free Trade Association.

The mandate required:

- harmonization of standards in Europe;
- inclusion of modern design methods;
- consideration of software practice.

CONCLUSIONS

The sandwich method is the result of half a century of evolution of scientific thought and engineering practice, which has finally received official recognition in the most important European standard for concrete structures.

The inclusion of this method in EN 1992-1-1:2023 became possible thanks to:

1) 50 years of successful application (1974-2023);
2) widespread use in software;
3) need for standardization for shells and slabs;
4) solid experimental database (dozens of studies);
5) simplicity and practicality for engineers;
6) conservatism (safety) of the method;
7) requirement of Mandate M/515 of the European Commission;
8) the scientific consensus reached (fib Model Code 2010).

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ІСТОРІЯ ВИНИКНЕННЯ ТА РОЗВИТКУ СЕНДВІЧ-МОДЕЛІ ДЛЯ РОЗРАХУНКУ МЕМБРАННИХ, ОБОЛОНКОВИХ ТА ПЛИТНИХ ЕЛЕМЕНТІВ ЗГІДНО З EN 1992-1-1:2023

Леонід СКОРУК

Анотація. Вперше в європейських нормах проектування бетонних конструкцій, зокрема в Додатку G (обов'язковий), у новій версії Єврокоду EN 1992-1-1:2023 приділено прицільну увагу проектуванню мембранних, оболонкових та плитних елементів та пропонується сучасний підхід для їх розрахунку.

У якості методу оптимального розрахунку залізобетонних оболонок, плит та мембран в Додатку G пропонується, так звана, **сендвіч-модель** – коли оболонка представляється як тришарова модель з двох несучих шарів (верхній і нижній) та проміжного шару між ними.

При розрахунку за сендвіч-моделлю – просторова задача (згин + крутіння + мембранні зусилля) перетворюється у дві мембранні задачі для верхнього та нижнього шарів. Тобто перетворення просторового напруженого стану на два шари (верхній і нижній) з еквівалентними мембранними напруженнями.

Модель сендвіча – це строга механічна модель, яка дозволяє звести складну просторову задачу (оболонка з комбінованими зусиллями) до двох незалежних плоских задач (мембранні елементи у верхньому та нижньому шарах) через статично еквівалентне перетворення зусиль у напруження.

Сендвіч-модель базується на перетворенні комбінації зусиль (мембранних, згинальних і крутних) у статично еквівалентну систему плоских напружень, які діють у верхньому та нижньому шарах моделі.

Основні припущення методу:

а) тришарова модель: оболонка представляється як конструкція з двох несучих шарів (верхній і нижній) та проміжного шару між ними

б) статична еквівалентність: внутрішні зусилля перетворюються у плоскі напруження так, щоб зберегти повну статичну рівновагу

в) незалежний розрахунок шарів: кожен шар (верхній і нижній) розраховується окремо як мембранний елемент згідно п. G.3

Формулювання, що представлені в Додатку G EN 1992-1-1:2023 у пунктах G.3 та G.4, узгоджуються з пунктами та положеннями щодо проектування у розділі 8 (Граничні стани міцності (ULS)) основного тіла документу, а пункт G.5 містить додаткові положення до 9.2 (Контроль тріщин) зазначених норм. Додаток G охоплює проектування плоских елементів без розривів бетонного масиву на окремі частини. При цьому можуть бути використані інші, більш вдосконалені методи розрахунку, що відповідають п.7.3.3 (Пластичний аналіз) або п.7.3.4 (Нелінійний аналіз) Єврокоду 2.

Таким чином, Додаток G (Проектування мембранних, оболонкових та плитних елементів) не замінює, а доповнює основні розділи Єврокоду 2, а саме – загальні принципи, матеріали (діаграми σ-ε), розрахунок перерізів на ULS, тріщиностійкість та SLS

Ключові слова: розрахунок мембранних, оболонкових та плитних елементів; сендвіч-модель

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