DOI: 10.32347/2522-4182.17.2025.17-29
UDC 624.012.45

IMPACT OF BLAST WAVE ARRIVAL DELAY ON THE DYNAMIC BEHAVIOR OF PROTECTIVE STRUCTURES

Anatoly PERELMUTER[1], Taliat AZIZOV[2], Dmytro KOCHKAREV[3], Nataliia SRIBNIAK[4]

[1]LLC SCAD Soft
24-B Zhukova St, Kyiv, Ukraine, 02166
[2,4]Sumy National Agrarian University
160 Herasym Kondratiev Street, Sumy, Ukraine, 40021
[3]National University of Water and Environmental Engineering
11 Soborna St., Rivne, Ukraine, 33028
[1]avp@scadsoft.com, https://orcid.org/0000-0001-9537-2728
[2]azizov.taliat@snau.edu.ua, https://orcid.org/0000-0001-9621-9805
[3]d.v.kochkarev@nuwm.edu.ua, https://orcid.org/0000-0002-4525-7315
[4]nataliia.sribniak@snau.edu.ua, https://orcid.org/0000-0003-3205-433X

Abstract. The article examines methods for calculating protective structures under the action of a blast wave, the pressure of which reaches different points at different times. A phase-by-phase calculation approach is considered, in which the action of a series of forces with different arrival times is treated as separate phases of vibration. The initial conditions of the current phase are taken as the final conditions (displacements and velocities) of the previous phase.

The study focuses in detail on a single-degree-of-freedom (SDOF) system subjected to forces with varying arrival times. At each phase, the constants of the particular and general solutions of the SDOF differential equation are determined based on the known right-hand side of the differential equation. Once all constants are obtained, the displacements at each phase are calculated.

It is shown that the advantage of this approach lies in the fact that, regardless of the number of intervals during which forces act at different times, only one differential equation with its own initial conditions and force set is solved at each phase. Consequently, the constants for the particular and general solutions are determined for each phase in dependently. Therefore, the number of intervals can be arbitrarily chosen by the engineer, and there is no added complexity in the numerical implementation even for systems with multiple forces.

It is demonstrated that for a single-mass system, different arrival times of the dynamic force do not increase the system response. However, this scheme is used when a multi-degree-of-freedom (MDOF) system is analyzed via modal decomposition, treating each mode as an SDOF system with consideration of the different force arrival times, and then summing the responses.

Anatoly PERELMUTER
Head of Research and Development, SCAD Software,
D.Sc. (Eng.)

Taliat AZIZOV
Professor, Department of Building Structures,
D.Sc. (Eng.)

Dmytro KOCHKAREV
Professor, Department of Urban Construction and Public Utilities,
D.Sc. (Eng.)

Nataliia SRIBNIAK
Associate Professor, Department of Building Structures,
PhD (Eng.)

It is shown why, in linear systems, summing the modal responses at specific times is correct, whereas simply summing the maximum responses is not.

The study confirms that the phase-by-phase consideration of system vibrations is correct, though more cumbersome. For multi-mass systems under impulsive loading, an analytical formula for the total response to impulses reaching different points at different times is provided. Both a triangular impulse with finite duration and an instantaneous impulse are considered.

Keywords: blast wave; vibration phase; single-degree-of-freedom (SDOF) system; equation of motion; impulse.

INTRODUCTION

Protective structures play an important role in safeguarding the population during air attacks. When conventional munitions are employed, protective structures must shield occupants from the blast wave and from fragmentation. This necessitates performing specific calculations, namely:

• the analysis of the load-bearing and enclosing elements of protective structures under the action of a blast shock wave;

• the analysis of the enclosing elements of protective structures under the action of fragments.

It should be noted that there are also certain types of structures that are designed to withstand the direct impact of individual munitions. As a rule, these are specialized facilities of critical importance. Additional, specialized calculations are performed for such structures, which are not considered in this article.

The structural analysis of the enclosing elements of protective buildings and facilities under blast-wave loading may be performed using one of three methods:

• the direct integration method of the equations of motion;

• the impulse (shock-impulse) method;

• the quasi-static method.

Each of these methods is characterized by its own type of loading. Recently, attention has also been drawn to the question of whether the non-uniform (non-simultaneous) arrival of blast-wave pressure affects the dynamic displacements and internal forces in protective-structure components.

ANALYSIS OF PREVIOUS RESEARCH

A considerable number of studies have been devoted to the investigation of blast wave effects. One of the key issues is the analysis of the shape and function of blast wave pressure [2, 12, 25]. In calculations, including those specified in regulatory documents, both curvilinear and simplified linear functions are used [5, 23, 24]. Additionally, computational tools for determining blast wave parameters are also available [13].

Research has shown that accounting for the negative phase of pressure leads to changes in dynamic forces in elements of protective structures [1, 14]. However, in many regulatory documents, calculations are performed considering only the positive phase of blast wave pressure.

There is extensive discussion and debate regarding the choice of calculation method: the direct integration of motion equations, the impulse method, and the quasi-static method. In Ukrainian standards [5], the quasi-static method is adopted. This approach is the simplest, though the least precise, but it allows relatively rapid determination of results. In the US standards [23, 24], the choice among direct integration, impulse, or quasi-static methods depends on the ratio of the positive phase duration of the blast to the natural period of structural vibration. Experimental and numerical studies [9, 17, 26, 27] have demonstrated the influence of various factors on the resistance of reinforced concrete and steel structures to blast wave action [7, 8, 16, 19]. The effect of damping devices on the system response to blast waves has also been investigated, considering different models and various pressure waveform shapes [3, 6,11, 20].

In many calculation methods, complex multi-degree-of-freedom systems are reduced to a single-degree-of-freedom system, and the solution of a known differential equation is considered, the right-hand side of which depends on the adopted blast wave pressure profile.

It is known that, although the duration of blast wave action is extremely short [2, 12, 13, 25], the pressure does not reach different points of the structure simultaneously. The extent to which this affects the response of the dynamic system is an important consideration for accurate structural analysis.

PURPOSE AND METHODS

An important and theoretically underexplored issue is the investigation of the influence of differing arrival times of blast-wave pressure at various points of a structure, which may affect the displacements and internal forces in structural elements either positively or negatively. It is necessary to determine which analytical method should be used for structural assessment in such cases. In view of the above, the aim of this article is to analyze the effect of blast-pressure arrival time at different points of a structure and to develop a methodology for performing such calculations.

The study employs a comprehensive set of methods aimed at an in-depth analysis of the behavior of protective structures subjected to blast loading that reaches different points at different times. The core of the work is an analytical approach to solving the differential equations of motion describing the vibrations of an SDOF system. For each time interval in which a force with a distinct arrival time acts, a phase-by-phase calculation method is applied: the equation of motion is solved separately for each phase, with the final conditions of the preceding phase—displacement and velocity—used as the initial conditions for the subsequent one. This enables a sequential and accurate determination of all constants of the particular and general solutions and provides a complete time-dependent response of the system regardless of the number of loading intervals.

For multi-degree-of-freedom systems, modal analysis is employed, allowing the complex system to be represented as a set of independent SDOF models. For each mode, the response is determined with consideration of the differing force arrival times, after which the modal displacements are superposed in the time domain. The study provides a detailed justification for why time-domain summation of modal responses is valid for linear systems, while summation of the modal peak values leads to erroneous results.

Additionally, a comparative analysis of results obtained using different approaches is performed, allowing the assessment of how temporal mismatch in force application influences the magnitude of the structural response. For impulsive loads, analytical expressions are derived for the total response to a series of impulses arriving at different points at different times. When necessary, the analytical solutions can be validated through direct numerical integration of the equations of motion, providing a computational verification of the accuracy of the proposed methodology.

This integrated methodological framework enables a thorough investigation of the influence of non-simultaneous blast-load arrival on structural systems and establishes a universal approach for their analysis and design.

MAIN PART

Let a vertical cantilever be given, with n lumped masses attached to it. It is known that such a multi-mass system can be reduced to a system with a single equivalent mass [2, 3, 16]. Let us consider this vertical cantilever of height L as a single-degree-of-freedom system with an equivalent mass $m = m_e qv$ at its free end.

A force $F_1(t) = P_1(1 - t/\tau)$ acts on it at a height a above ground level, and a force $F_2(t) = P_2(1 - t/\tau)$ acts at a height b, where force F_1 begins to act at time $t = 0$, while force F_2 is applied with a delay, at time $t = \delta_t$ (Fig. 1).

Given that the distance from the explosion epicenter to different points along the height of a structure varies, it is necessary to investigate the extent to which the non-simultaneous arrival of the blast wave affects the stress–strain state of the system.

ISSN 2522-4182

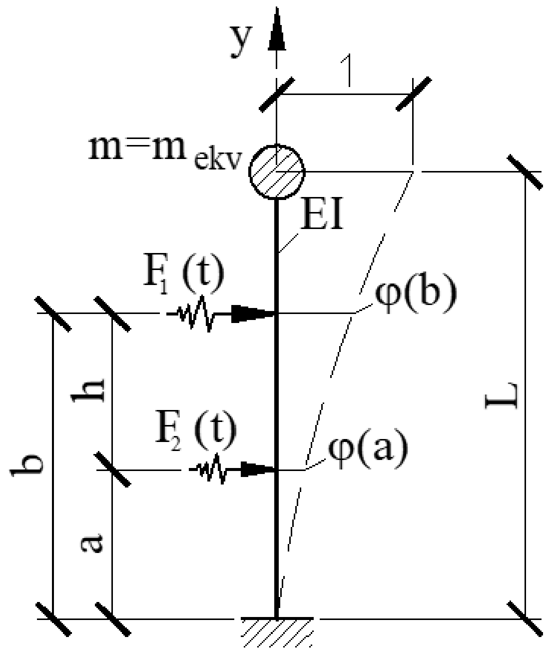

Fig. 1 SDOF system diagram with differential arrival of dynamic forces
Рис. 1 Схема системи з однією масою з різним приходом динамічних сил

If the cantilever is a homogeneous rod, the equivalent mass is known to be determined by the expression $m_e qv = 0.243 \cdot m_{tot}$,

where m_{tot} is the total mass of the rod.

Several remarks should be made here.

1.A specified pressure is considered, and we do not address which specific type of pressure generates the forces F_1 and F_2—whether incident, reflected, etc.—because our primary objective is to determine whether the delay in force application has an influence or not.

2.We analyze a simple scheme with two forces, because such a simplified model allows us to clearly demonstrate this effect numerically. The essence of the analysis does not change when a different number of forces is considered.

Let us perform the analysis on the basis of the following theoretical approach.

The well-known differential equation without damping takes the form:

$$m \cdot \ddot{x} + k \cdot x = Q(t) \quad (1)$$

where $x = x(t)$ is the generalized coordinate, taken here as the horizontal displacement of mass m; k is the stiffness coefficient of the cantilever, which in our case is $k = 3 \cdot EJ / L^3$,

where EJ is the bending stiffness of the rod; and $Q(t)$ is the generalized force (the coefficient corresponding to the virtual displacement of the generalized coordinate).

The delay of the second force, δ_t, will be considered smaller than the duration of the positive phase of the blast wave, τ, as will be briefly discussed below. At the beginning of the calculation, the value of δt can be chosen such that $\tau = n \cdot \delta_t$. Let us illustrate this with an example where $n = 4$. In this case, several phases of vibration must be considered:

Phase 1:

$0 \le t \le \delta_t$ — only force F_1 acts; $F_2 = 0$.

Phase 2:

$\delta_t \le t \le \tau$ — both forces F_1 and F_2 act; however, the second force begins at $t = \delta_t$.

Phase 3:

$\tau \le t \le \tau + \delta_t$ — only force F_2 acts;

$F_1 = 0$.

Phase 4:

$t \ge \tau + \delta_t$ — free vibration phase;

$F_1 = F_2 = 0$.

Taking into account the definition of the generalized force $Q(t)$, the differential equation of motion (1) for all phases can be written as follows:

$$\ddot{x}+\omega^2\cdot x=\frac{1}{m}\left[P_1\left(1-\frac{t}{\tau}\right)\varphi(a)+P_2\left(1-\frac{t}{\tau}\right)\varphi(b)\right] \quad (2)$$

where, as is known, ω is the circular frequency; *φ(a)* and *φ(b)* are the normalized displacements at points *a* and *b*, corresponding to the application points of forces F_1 and F_2, respectively, which are determined from the assumed bending shape of the cantilever.

$$\varphi(\mathrm{y})=\frac{y^2\left(3L-y\right)}{2L^3} \quad (3)$$

where *y* is the coordinate along the rod (see Fig. 1). Function (3) is chosen such that at
$y = L$ we have $\varphi(y) = 1$, and it corresponds to the bending shape of a simple cantilever beam.

The general solution of the equation of motion in each phase ($i = 1\ldots4$) is assumed in the form:

$$x_i(t)=C_{i,1}Cos(\omega\cdot t)+C_{i,2}Sin(\omega\cdot t)+A_i+B_i t \quad (4)$$

The particular solutions are expressed as linear functions with undetermined coefficients:

$$x_i^*(t)=A_i+B_i\cdot t \quad (5)$$

The constants A_i and B_i are determined depending on the right-hand side of expression (2) in each phase of vibration (see above). In the fourth phase, the constants A_4 and B_4 are absent. The constants $C_{i,1}$ and $C_{i,2}$ are determined from the initial conditions of each phase:

Phase 1: initial conditions are zero:

$$x_1(0)=0;\quad \dot{x}_1(0)=0$$

Phase 2: initial conditions are:

$$x_2(\delta_t)=x_1(\delta_t);\quad \dot{x}_2(\delta_t)=\dot{x}_1(\delta_t)$$

Phase 3: initial conditions are:

$$x_3(\tau)=x_2(\tau);\quad \dot{x}_3(\tau)=\dot{x}_2(\tau)$$

Phase 4: initial conditions are:

$$x_4(\tau+\delta_t)=x_3(\tau+\delta_t);\qquad \dot{x}_4(\tau+\delta_t)=\dot{x}_3(\tau+\delta_t).$$

That is, the initial conditions for the *i*-th phase are taken equal to the final values of displacement and velocity from the ($i-1$)-th phase, for which the solution has already been obtained.

The problem is solved according to the following algorithm:

In each phase, first, depending on the right-hand side of (2) and assuming a particular solution in the form of (5), the constants A_i and B_i are determined by equating coefficients of like powers of *t*. Then, using the general solution with the right-hand side (4) and applying the initial conditions (see above), the constants $C_{i,1}$ and $C_{i,2}$ are determined. Having all constants, all displacement values in each phase are calculated. The initial conditions for the next phase are the final conditions (displacement and velocity) of the previous phase. This procedure is repeated until the end of the fourth phase.

For a larger number of forces $F_1, F_2, \ldots F_n$, as well as for different durations of the positive phase τ_i of these forces, the essence of the calculation does not change. In this case, the differential equation (2) takes the form:

$$\ddot{x}+\omega^2\cdot x=\frac{1}{m}\left[P_1\left(1-\frac{t_1}{\tau_1}\right)\varphi(y_1)+\ldots+P_n\left(1-\frac{t_n}{\tau_n}\right)\varphi(y_n)\right] \quad (6)$$

where $y_1, \ldots, y_n$ are the distances from the base of the cantilever to the force F_n. In this case, the equation (6) should be considered sequentially; t_i is the time at the beginning of the i-th segment of the analysis of equation (6), which is determined by the formula:

$$t_i = t-(i-1)\delta_t \quad (7)$$

That is, for each time segment, a new variable t_i is introduced. In this case, the right-hand side of equation (6) will include the number P_i depending on the arrival time t_i of force F_i, the durations of forces $F_1, F_2, \ldots F_{i-1}$, as well as times $t_{i+1}, t_{i+2}, \ldots t_n$.

The number of considered phases depends on the number of segments dividing the total time, $t_{tot} = n\cdot\boldsymbol{\delta_t}$. The initial conditions for each *i*-th phase are taken as the final conditions (displacement and velocity of the mass) at the end of the ($i-1$)-th phase.

It should be particularly noted that, regardless of the number of segments, only one differential equation (6) is solved in each phase, with its own initial conditions and its own set of forces F_i. Therefore, the constants A_i and B_i from the particular solution of the equation (see expression 5) are determined each time. As a result, the number of segments can be chosen arbitrarily by the engineer, and there is no particular difficulty in the numerical implementation of the calculation for a system with many applied forces.

The problem is significantly simplified if the loading is considered as an instantaneous impulse. Then, if n impulses $J_1, J_2, \ldots J_n$ act sequentially, each applied after a time interval $\boldsymbol{\delta_t}$, the differential equation takes the form of equation (1) with a zero right-hand side. The solutions of these equations for the action of the i-th impulse are expressed as:

$$x_i(t) = \frac{J_i \cdot \phi_i}{m \cdot \omega} Sin(\omega \cdot t); \quad \dot{x}_i(t) = \frac{J_i \cdot \phi_i}{m} Cos(\omega \cdot t) \qquad (8)$$

The difference for all the equations lies in the initial conditions. Thus, for two impulses, the initial conditions for the first impulse are $x(0) = 0;\ \dot{x}(0) = J_1\phi(a)/m$, where $\varphi(a)$ is determined from expression (3). The initial conditions for the second impulse are:

$$x(\delta_t) = \frac{J_1\phi(a)}{m \cdot \omega} Sin(\omega \cdot \delta_t); \dot{x}(\delta_t) = \frac{J_1\phi(a)}{m \cdot \omega} Cos(\omega \cdot \delta_t) + \frac{J_2\phi(b)}{m} \qquad (9)$$

Under the action of n impulses, the initial conditions for the n-th impulse are as follows:

$$x(n \cdot \delta_t) = \frac{J_1\phi_1}{m \cdot \omega} Sin(\omega \cdot \delta_t) + \frac{J_1\phi_2}{m \cdot \omega} Sin(\omega \cdot 2 \cdot\cdot \delta_t) + \ldots + \frac{J_n\phi_n}{m \cdot \omega} Sin(\omega \cdot n \cdot \delta_t) \qquad (10)$$

$$\dot{x}(n \cdot \delta_t) = \frac{J_1\phi_1}{m \cdot \omega} Cos(\omega \cdot \delta_t) + \cdots + \frac{J_{n-1}\phi_{n-1}}{m \cdot \omega} Cos(\omega \cdot (n-1) \cdot \delta_t) + \frac{J_n\phi_n}{m} \qquad (11)$$

where $\varphi_1, \ldots, \varphi_n$ denote the functions (3) corresponding to the locations of the *1st* … *n*-th impulses.

Thus, at the moment the impulse with index k is applied, the initial displacement is equal to the sum of the displacements of the free vibrations from impulses $1, \ldots, k{-}1$ according to the first expression in (8), and the initial velocity is equal to the sum of the velocities from these impulses according to the second expression in (8), plus the initial velocity generated by the k-th impulse itself.

It is known that if the system is considered not as one with a single equivalent mass but as one with the actual number of masses equal to n, then after modal analysis one can obtain n separate differential equations of type (1). Solving them yields a set of expressions $x_i(t)$ for each mode of vibration. The total response of the system (for example, the total displacement of the i-th mass) is simply the sum of the displacements $x_i(t)$ for all modes at the considered time t.

Reducing a system with n masses to a system with a single equivalent mass, as discussed above, provides an approximate solution; however, it allows the effect of delayed force arrival to be taken into account.

Calculations show that, for a single-mass system, the delay in force application does not increase the system’s response. However, for multi-mass systems, this effect may either increase or decrease the total response. The analysis of multi-mass systems can be performed using the methodology developed above, but with the application of modal decomposition, where each mode is treated as an SDOF system while still accounting for the delayed arrival of forces.

ISSN 2522-4182

The methodology proposed above involves a multiphase treatment of the problem. The use of the Duhamel integral for solving the equations of motion with forces applied at different times makes it unnecessary to consider initial conditions at each phase. Let us examine an approach for analyzing a multi-mass system subjected to impulses applied with time shifts τ_i, using the Duhamel integral. Since the response maxima occur at different moments in time, a simple summation of these maxima sum not the maxima, but the modal responses evaluated at the same time instant *t*. For the mode with index *r*, we obtain the following differential equation:

$$m_r \ddot{\eta}_r + k_r \eta_r = F_r(t), \qquad (12)$$

where $\eta_r = \eta_r(t)$ is the generalized coordinate in the r-th mode, determined by the well-known Duhamel integral:

$$\eta_r(t) = \frac{1}{m_r \omega_r} \int_0^t F_r(\tau) sin(\omega_r(t-\tau)) d\tau. \qquad (13)$$

where m_r is the modal mass;
$k_r = m_r \omega_r^2$ is the stiffness in the *r*-th mode.

$$F_j(\tau) = p(t-\tau_j) A_j \qquad (14)$$

where *p(t)* is the triangular impulse generated by the blast wave; τ_j is the arrival time of the wave at node *j*; A_j is the area from which the load is collected for that node.

$$p(t) = \begin{cases} p_0\left(1-\dfrac{t}{t_+}\right), 0 \le t \le t_+ \\ 0, t > t_+. \end{cases} \qquad (15)$$

where p_0 is the amplitude; t_+ is the duration of the impulse.

Substituting into the Duhamel integral for a single node j gives us the expression:

$$\eta_{rj}(t) = \frac{p_0\, A_j \varphi_r(x_j)}{m_r \omega_r} \int_{\tau_j}^t \left(1-\frac{t-\tau_j}{t_+}\right) \times sin(\omega_r(t-\tau)) d\tau \qquad (16)$$

Let us introduce a new variable:

$$u=\tau-\tau j,\ d\tau=du \qquad (17)$$

Then we will have:

$$\eta_{rj}(t) = \frac{p_0\, A_j \varphi_r(x_j)}{m_r \omega_r} \int_0^{u*} (1-\frac{u}{t_+})\, sin(\omega_r(t-\tau_j) - \omega_r u) d\tau \qquad (18)$$

where $u*=min(t-\tau_j, t+)$; is introduced;
φ_r –is the *r*-th mode shape.

In addition, let us denote $\alpha=\omega_r(t-\tau_j)$. Then the integral can be written as follows:

$$J(\alpha, u^*) = \int_0^{u*} (1-\frac{u}{t_+}) sin(\alpha - \omega_r u) du \qquad (19)$$

After expanding the integrand, applying integration by parts, imposing the limits from *0* to *u**, and performing the necessary transformations, we finally obtain:

$$J(\alpha, u^*) = \frac{1}{\omega}\, [\cos(\alpha - \omega \cdot u^*) - cos\alpha] - \frac{1}{t_+}(-\frac{u^*}{\omega} \cos(\alpha - \omega \cdot u^*) + + \frac{1}{\omega^2} sin(\alpha - \omega \cdot u^*) - \frac{1}{\omega^2} sin\alpha) \qquad (20)$$

Then the final formula for the response in the *r*-th mode takes the form:

$$\eta_r(t) = \frac{1}{m_r\omega_r}\sum p_0 A_j \varphi_r(x_j) \cdot J(\omega_r(t - -\tau_j), u_j^*) \quad (21)$$

Let us now explain the reason why the responses can be summed. Solution (12) is presented in the form of (13). In this case, the modal force is determined as the sum over the segments:

$$F_r(t) = \sum_{i=1}^{N} P_{rj} S\big(t - \tau_j\big), \quad (22)$$

where $P_{rj} = p_0 A_j \phi_r(x_j)$.

From the linearity of the integral, we have:

$$\eta_r(t) = \sum_{i=1}^{N} \frac{P_{rj}}{m_r\omega_r} \int_0^t S(\tau - \tau_j) \times sin(\omega_r(t - \tau))d\tau. \quad (23)$$

Let us make the substitution $u=\tau-\tau_j$ and obtain the shifted response function:

$$\eta_r(t) = \sum_{i=1}^{N} \frac{P_{rj}}{m_r\omega_r} J_r(t - \tau_j), \quad (24)$$

where:

$$J_r(\xi) = \int_0^{min(\xi,t+)} S(u) \cdot sin(\omega_r(\xi - u))du, \quad (25)$$

$$J_r(\xi) \equiv 0 \text{ при } \xi < 0 \quad (26)$$

Here, $J_r(\xi)$ is a time-dependent function. It is zero before arrival and then varies according to a sine function.

We sum the values of these functions at the same time *t*:

$$\eta_r(t) = \sum \frac{P_{rj}}{m_r\omega_r} J_r(t - \tau_j) \quad (27)$$

This summation is correct for a linear system.

In the considered method, *J* is not a number, but a time-dependent function (a convolution kernel). To avoid interpreting *J* as a number, we fix the notation as follows:

$$\eta_r(t) = \sum_{i=1}^{N} \frac{p_0 A_j \phi_r\big(x_j\big)}{m_r\omega_r} J_r\big(t - \tau_j\big),$$

$$J_r(\xi) = H(\xi) \int_0^{min(\xi,t+)} S(u) \sin(\omega_r(\xi - u))$$

where $H(\xi)$ is the well-known unit Heaviside function. For a very short pulse $\omega_r t_+ \ll 1$, for $Jr(\xi)$ we obtain:

$$J_r(\xi) \approx \frac{t_+}{2} sin(\omega_r \xi) H(\xi) \quad (28)$$

and then we have:

$$\eta_r(t) = \sum_{j=1}^{N} \frac{P_{rj}}{m_r\omega_r} \frac{t_+}{2} sin(\omega_r(t - \tau_j)) \quad (29)$$

$$\eta_r(t) = \sum_{j=1}^{N} \frac{P_{rj}}{m_r\omega_r}\frac{t_+}{2}\sin\left(\omega_r\left(t-\tau_j\right)\right)\eta_r(t) = \frac{1}{m_r\omega_r}P_r sin(\omega_r t) \tag{30}$$

Thus, in the form (29) we obtained the total response of the system without performing a phase-by-phase computation, as was shown above. This is the advantage of such an approach. However, for calculations that take the system's nonlinearity into account, one should still use the phase-by-phase procedure, in which, as shown earlier, the initial conditions of each subsequent phase are taken as the final conditions of the preceding phase of oscillations.

Example of Calculation.

A vertical three-story cantilever beam is considered, with lumped masses *m=3000 kg* at each floor level. The applied forces are arranged as shown in Fig. 2. The cross-section is $b\times h=0.5\times1$ *m*, the modulus of elasticity is *E=25,000 MP*, and the TNT charge mass is *W=100 kg*. For the first force, the following parameters are adopted: $P_1=1184.67$ *kN*, $t_{01}=0.00715$ *s* (the duration of force P_1), $t_{A1}=0$ *s* (arrival time of P_1). For the second force, the parameters are: $P_2=725.25$ *kN*, $t_{02}=0{,}00737$ *s* (the duration of the force P_2), $t_{A2}=0.001$ *s* (arrival time of P_2). For the third force, the adopted parameters are: $P_3=303.66$ *kN*, $t_{03}=0{,}00781$ *s* (the duration of force P_3), $t_{A3}=0.004$ *s* (arrival time of P_3).

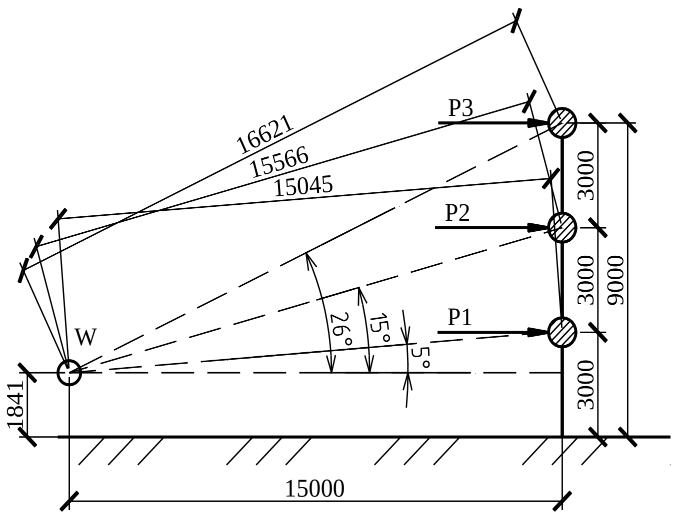

Fig. 2 Diagram of a single-mass system subjected to dynamic forces with different arrival times
Рис. 2 Схема системи з однією масою з різним приходом динамічних сил

Solution. The fundamental vibration period of the cantilever beam shown in Fig. 2 is *T=0.191 s*. The arrival times of forces P_2 and P_3 are $t_{A2}=0.001$ *s<< T= 0.191 s* and $t_{A3}=0.004$ *s< <<T = 0.191 s* respectively. Therefore, the internal force values in the beam for the case of simultaneous arrival of the forces and for the case with the specified delays will be practically identical.

In the analysis of the beam under the assumption of simultaneous force arrival, the computed maximum bending moment at the fixed support is $M_{max}=280$ *kN·m*. When the delay in force application is taken into account using the proposed methodologies, the maximum bending moment is obtained as *283.26 kN·m,* corresponding to a relative error of *1.15%.*

When the force arrival times are increased to $t_{A2}=0.004$ *s* and $t_{A3}=0.008$ *s*, the calculations yield a bending moment of *341.27 kN·m* , with a relative error of *21.88%.*

Further increasing the arrival times to $t_{A2}=0.008$ *s* and $t_{A3}=0.016$ *s* results in a bending moment of *310.56 kN·m*, corresponding to a relative error of *10.9%.*

For the arrival times $t_{A2}=T/6=0.0318$ *s* and, $t_{A3}=T/3=0.0637$ s, the calculations yield a bending moment of *270.87 kN·m,* corresponding to a relative error of *–3.37%.*

For the arrival times $t_{A2}=T/3=0.0637$ *s* and $t_{A3}=T/2=0.0955$ *s,* the bending moment is 251.45 *kN·m,* with a relative error of *–11.14%.*

Thus, we have demonstrated that accounting for delayed force arrival on subsequent floors may lead to either an increase or a decrease in the dynamic internal forces. This effect depends on factors such as the duration of the positive phase of the pressure, the delay time of the forces, and the vibration period of the system.

CONCLUSIONS AND RECOMMENDATIONS

The proposed methodology for analyzing structural response under blast loading accounts for the non-simultaneous arrival of the blast wave at different points of the structure. The maximum response may occur during different phases of the mass motion. A phase-by-phase analysis is presented, in which the initial conditions of each subsequent phase are taken as the final conditions of the previous vibration phase. An analytical expression for the total response under multiple impulses acting on different masses of the system at different times is also derived. It is demonstrated why, in linear systems, the superposition of modal responses at a specific moment of vibration is valid. For SDOF systems, the influence of force delay has a minor effect on the system's response.

However, for multi-mass systems, the effect of delayed forces may lead either to an increase or a decrease in the overall response, depending on the ratio t_0/T and the delay time.

REFERENCES

1. **Azizov T.N., Kochkarev D.V., Galinska T.A.** (2025). Buildings and Structures Calculations for Air Shock Wave Effect from Conventional Weapons. *IOP Conference Series: Earth and Environmental Science*, 1499(1), 012012. https://doi.org/10.1088/1755-1315/1499/1/012012
2. **Baker W.E.** (1973). Explosions in Air. Univ. of Texas Press, Austin, TX, USA, 268 *pp.*
3. Blast effects on buildings. Design of buildings to optimize resistance to blast loading. (1995). *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 32(7), 348–349.* https://doi.org/10.1016/0148-9062(95)92584-5
4. **Clough R.W., Penzien J. (2015).** Dynamics of Structures. *New Delhi: McGraw-Hill Education, 319..*
5. **DBN V.2.2-5:2023. (2023).** Tsivilni zakhysni konstruktsii [Civil defense protective structures]. *[Chynnyi vid 1.11.2023],* Kyiv: *Ministerstvo rozvytku hromad, terytoriy ta infrastruktury Ukrayiny, 122 s. Redaktsiya stanom na 25.12.2024..* [in Ukrainian]
6. **Dennis A.A., Rigby S.E.** (2024). The direction-encoded neural network: a machine learning approach to rapidly predict blast loading in obstructed environments. *Int. J. Protect. Struct., 15(3), 455–483.* https://doi.org/10.1177/20414196231177364
7. **Task Committee on Blast Resistant Design of the Petrochemical Committee of the Energy Division of the American Society of Civil Engineers.** (2010). *Design of blast-resistant buildings in petrochemical facilities. pp. 447*
8. **Ekström J., Rempling R., Plos M.** (2016). Spalling in concrete subjected to shock wave blast. *Engineering Structures, 122, 72–82.* https://doi.org/10.1016/j.engstruct.2016.05.002
9. **Fan Y., Chen T., Yang G., Cui X., Lu W., Wang G., Tian B.** (2024). Experimental investigation on dynamic response of concrete gravity dam under shock wave and bubble pulsation. *Engineering Structures, 318, 118796*. https://doi.org/10.1016/j.engstruct.2024.118796
10. **Isaac O.S., Alshammari O.G., Pickering E.G., Clarke S.D., Rigby S.E.** (2023). Blast wave interaction with structures—an overview. *Int. J. Protect. Struct., 14(4), 584–630*. https://doi.org/10.1177/20414196221118595
11. **Jin R., Gou Y**. (2018). Motion response analysis of large-scale structures with small-scale cylinders under wave action. O*cean Engineering, 155, 65–74.* https://doi.org/10.1016/j.oceaneng.2018.02.032
12. **Karlos V., Solomos G.** (2013). Calculation of blast loads for application to structural

components. *JRC Technical Report, EUR 26456 EN, 58 pp.* https://doi.org/10.2788/61866

13. **Kingery C.N., Bulmash G**. (1984). Technical report ARBRL-TR-02555: Air blast parameters from TNT spherical air burst and hemispherical burst. A*D-B082 713, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 51 p.*
14. **Kochkarev D., Azizov T., Galinska T**. (2024). Calculation of Buildings and Structures for Air Blasts Using Explosion Accelerograms. *Lecture Notes in Civil Engineering, 604 LNCE, pp. 245–256.* https://doi.org/10.1007/978-3-031-67576-8_22
15. **Kochkarev D., Azizov T., Galinska T**. (2024). Calculation of Enclosures of Defence Structures Based on the Quasi-static Method. *Lecture Notes in Civil Engineering, 469 LNCE, pp. 50–58.* https://doi.org/10.1007/978-3-031-55068-3_4
16. **Koliakova V., Dumych A., Sumak A.** (2023). Stress-Strain State of Shelter Structures under the Action of Air Shock Wave. *Sworld-Us Conference Proceedings, (1) (usc22-01), 49–56*. [in Ukrainian] https://doi.org/10.30888/2709-2267.2024-22-00-020
17. **Kumar V., Kartik K.V., Iqbal M.A.** (2020). Experimental and numerical investigation of reinforced concrete slabs under blast loading. *Engineering Structures, 206, 110125.* https://doi.org/10.1016/j.engstruct.2019.110125
18. **Wang L., Kong D. (**2023). Influence of Ground Impedance on Explosive Shock Wave Test Accuracy. *International Journal of Impact Engineering, 171, 104395* https://doi.org/10.1016/j.ijimpeng.2022.104395
19. **Li Q., Wang Y., Li L., Hao H., Wang R., Li J**. (2023). Prediction of BLEVE loads on structures using machine learning and CFD. *Process Safety and Environmental Protection, 171, 914–925.* https://doi.org/10.1016/j.psep.2023.02.008
20. **Cormie D., Smith P.D., Mays G.C**. (2019). Blast Effects on Buildings: Design of Buildings to Optimize Resistance to Blast Loading. 3rd ed., *ICE Publishing / Emerald Publishing, 344 .*
21. **Moon K.** (2011). Structural Design of Double Skin Facades as Damping Devices for Tall Buildings. *Procedia Engineering, 14, 1351–1358.* https://doi.org/10.1016/j.proeng.2011.07.170
22. **Xiao S.Y., et al**. (2021). Prediction of warhead explosion location based on explosion shock wave. *Military Automation, 40(5), 90–93.* https://doi.org/ 10.7690/bgzdh.2021.05.020
23. **U.S. Department of Defense.** (2018, change 3 — 24 May 2024). UFC 4-010-01, DoD Minimum Antiterrorism Standards for Buildings. *Washington, DC. pdf, 95*
24. **U.S. Army Corps of Engineers, Naval Facilities Engineering Command, Air Force Civil Engineer Support Agency**. (2014, Change 2 — 01 September 2014). UFC 3-340-02, Structures to Resist the Effects of Accidental *Explosions. Washington, DC. pdf, 99*
25. **Karlos V., Solomos G.** (2013). Calculation of blast loads for application to structural components. *Luxembourg: Publications Office of the European Union, 58 .* https://doi.org/ 10.2788/61866
26. **Vivek P., Sitharam T.G.** (2019). Response of Embedded Structures in Granular Material to Air-Blast Wave Loading. *In: Granular Materials Under Shock and Blast Loading, 95–109.* https://doi.org/10.1007/978-981-15-0438-9_6
27. **Wang Z., Gong X., Xiong J., Yong H.** (2014). Studying an engineering model on an air blast wave. Structures Under Shock and Impact XIII, 1, 217–227. https://doi.org/10.2495/susi140191

LITERATURE

1. **Azizov T.N., Kochkarev D.V., Galinska T.A.** (2025). Buildings and Structures Calculations for Air Shock Wave Effect from Conventional Weapons. *IOP Conference Series: Earth and Environmental Science*, 1499(1), 012012. https://doi.org/10.1088/1755-1315/1499/1/012012
2. **Baker W.E.** (1973). Explosions in Air. Univ. of Texas Press, Austin, TX, USA
3. Blast effects on buildings. Design of buildings to optimize resistance to blast loading. (1995). International Journal of Rock Mechanics and *Mining Sciences & Geomechanics Abstracts, 32(7), 348–349.* https://doi.org/10.1016/0148-9062(95)92584-5
4. **Clough R.W., Penzien J. (2015).** Dynamics of Structures. *New Delhi: McGraw-Hill Education, 319*
5. **DBN V.2.2-5:2023. (2023).** Tsivilni zakhysni konstruktsii. *Kyiv: Ministerstvo rozvytku hromad, terytoriy ta infrastruktury Ukrayiny,* 122 s. Redaktsiya stanom na 25.12.2024.
6. **Dennis A.A., Rigby S.E.** (2024). The direction-encoded neural network: a machine learning approach to rapidly predict blast loading in obstructed environments. *Int. J. Protect. Struct., 15(3), 455–483.*

https://doi.org/10.1177/20414196231177364

7.**Task Committee on Blast Resistant Design of the Petrochemical Committee of the Energy Division of the American Society of Civil Engineers.** (2010). *Design of blast-resistant buildings in petrochemical facilities.. 447*

8.**Ekström J., Rempling R., Plos M.** (2016). Spalling in concrete subjected to shock wave blast. *Engineering Structures, 122, 72–82.* https://doi.org/10.1016/j.engstruct.2016.05.002

9.**Fan Y., Chen T., Yang G., Cui X., Lu W., Wang G., Tian B.** (2024). Experimental investigation on dynamic response of concrete gravity dam under shock wave and bubble pulsation. *Engineering Structures, 318, 118796.* https://doi.org/10.1016/j.engstruct.2024.118796

10.**Isaac O.S., Alshammari O.G., Pickering E.G., Clarke S.D., Rigby S.E.** (2023). Blast wave interaction with structures—an overview. *Int. J. Protect. Struct., 14(4), 584–630.* https://doi.org/10.1177/20414196221118595

11.**Jin R., Gou Y**. (2018). Motion response analysis of large-scale structures with small-scale cylinders under wave action. *Ocean Engineering, 155, 65–74.* https://doi.org/10.1016/j.oceaneng.2018.02.032

12.**Karlos V., Solomos G.** (2013). Calculation of blast loads for application to structural components. *JRC Technical Report, EUR 26456 EN, 58.* https://doi.org/10.2788/61866

13.**Kingery C.N., Bulmash G**. (1984). Technical report ARBRL-TR-02555: Air blast parameters from TNT spherical air burst and hemispherical burst. *AD-B082 713, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 51.*

14.**Kochkarev D., Azizov T., Galinska T**. (2024). Calculation of Buildings and Structures for Air Blasts Using Explosion Accelerograms. *Lecture Notes in Civil Engineering, 604 LNCE, pp. 245–256.* https://doi.org/10.1007/978-3-031-67576-8_22

15.**Kochkarev D., Azizov T., Galinska T**. (2024). Calculation of Enclosures of Defence Structures Based on the Quasi-static Method. *Lecture Notes in Civil Engineering, 469 LNCE,. 50–58.* https://doi.org/10.1007/978-3-031-55068-3_4

16.**Koliakova V., Dumych A., Sumak A.** (2023). Napruzheno-deformovanyi stan konstruktsii ukryttia pry dii povitrianoi udarnoi khvyli *Sworld-Us Conference Proceedings,(1) usc22-01, 49–56.* https://doi.org/10.30888/2709-2267.2024-22-00-020

17.**Kumar V., Kartik K.V., Iqbal M.A.** (2020). Experimental and numerical investigation of reinforced concrete slabs under blast loading. *Engineering Structures, 206, 110125.* https://doi.org/10.1016/j.engstruct.2019.110125

18.**Wang L., Kong D. (**2023). Influence of Ground Impedance on Explosive Shock Wave Test Accuracy. *International Journal of Impact Engineering, 171, 104395.* https://doi.org/10.1016/j.ijimpeng.2022.104395

19.**Li Q., Wang Y., Li L., Hao H., Wang R., Li J**. (2023). Prediction of BLEVE loads on structures using machine learning and CFD. *Process Safety and Environmental Protection, 171, 914–925*. https://doi.org/10.1016/j.psep.2023.02.008

20.**Cormie D., Smith P.D., Mays G.C**. (2019). Blast Effects on Buildings: Design of Buildings to Optimize Resistance to Blast Loading. 3rd ed., *ICE Publishing / Emerald Publishing, 344 pp.*

21.**Moon K.** (2011). Structural Design of Double Skin Facades as Damping Devices for Tall Buildings. P*rocedia Engineering, 14, 1351–1358.* https://doi.org/10.1016/j.proeng.2011.07.170

22.**Xiao S.Y., et al**. (2021). Prediction of warhead explosion location based on explosion shock wave. *Military Automation, 40(5), 90–93.* https://doi.org/10.7690/bgzdh.2021.05.020

23.**U.S. Department of Defense.** (2018, change 3 — 24 May 2024). UFC 4-010-01, DoD Minimum Antiterrorism Standards for *Buildings. Washington, DC. pdf, 95*

24.**U.S. Army Corps of Engineers, Naval Facilities Engineering Command, Air Force Civil Engineer Support Agency**. (2014, Change 2 — 01 September 2014). UFC 3-340-02, Structures to Resist the Effects of Accidental *Explosions. Washington, DC. pdf, 99*

25.**Karlos V., Solomos G.** (2013). Calculation of blast loads for application to structural components. *Luxembourg: Publications Office of the European Union, 58 .* https://doi.org/202010.2788/61866

26.**Vivek P., Sitharam T.G.** (2019). Response of Embedded Structures in Granular Material to Air-Blast Wave Loading. In: *Granular Materials Under Shock and Blast Loading, pp. 95–109.* https://doi.org/10.1007/978-981-15-0438-9_6

27.**Wang Z., Gong X., Xiong J., Yong H.** (2014). Studying an engineering model on an air blast wave. *Structures Under Shock and Impact XIII, 1, 217–227.* https://doi.org/10.2495/susi140191

ВПЛИВ ЗАПІЗНЕННЯ ПРИХОДУ ВИБУХОВОЇ ХВИЛІ НА ДИНАМІЧНУ ПОВЕДІНКУ ЗАХИСНОЇ СПОРУДИ

Анатолій ПЕРЕЛЬМУТЕР
Талят АЗІЗОВ
Дмитро КОЧКАРЬОВ
Наталія СРІБНЯК

Анотація. В статті розглянуто методики розрахунку захисних споруд за дії вибухової хвилі, тиск якої приходить до різних точок в різний час. Розглянуто пофазовий спосіб розрахунку, коли дію серії сил з різним часом приходу розглядають як окремі фази коливань. За початкові умови поточної фази приймаються кінцеві умови (переміщення і швидкості) попередньої фази. Докладно розглянуто систему з однією масою (SDOF-систему), на яку діють сили з різним часом приходу. На кожній фазі спочатку в залежності від правої частини відомого диференціального рівняння знаходять константи частинного і загального рішення диференціального рівняння SDOF-системи. Маючі всі константи, визначають всі значення переміщень на кожній фазі.

Показано, що перевага такого підходу полягає в тому, що незважаючи на будь яку кількість ділянок, на які діють сили в різний час, на кожній фазі вирішується лише одне диференціальне рівняння зі своїми початковими умовами і своїм набором сил. Тому кожний раз знаходяться свої константи з частинного рішення та загального рішень рівняння. Тому кількість ділянок може бути визначена будь якою на думку інженера і якоїсь складності в чисельній реалізації розрахунку система з багатьма розглядуваними силами немає.

Показано, що для одномасової системи різний час приходи динамічної сили не збільшує відгук системи, але така схема використовується, коли багатомасову систему за допомогою модального розкладення розглядають як SDOF-систему для кожної окремої моди з врахуванням різного часу приходу сили, а потім складають відгуки простим підсумовуванням. Показано, чому в лінійних системах підсумовування модальних відгуків в конкретний час коливань є правильним, але просте підсумовування максимальних відгуків не є правильним.

Показано, що пофазовий розгляд коливань системи є правильним, але більш громіздким. Для багатомасових систем за дії імпульсу наведено аналітичну формулу сумарного відгуку на імпульси, які приходять до різних точок в різний час. При цьому розглянуто трикутний імпульс з кінцевим часом дії, а також миттєвий імпульс.

Ключові слова: вибухова хвиля; фаза коливань; SDOF-система; рівняння коливань; імпульс.

.

Received: *October 10, 2025.*
Accepted: *November 30, 2025*