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STRENGTH OF REINFORCED CONCRETE CONSTRUCTIONS UNDER PUNCHING SHEAR

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Abstract. Punching is one of the possible types of failure of reinforced concrete structures and occurs when a concentrated force is applied to slab slabs through an area commensurate with the height of the slab. In particular, when columns rest on transfer slabs, foundation slabs, and others.

Existing methods for calculating the strength of reinforced concrete slabs under punching are based on an empirical approach, which does not always adequately assess the strength of slabs and the influence of the main factors. At the same time, the general nature of failure under punching and the action of shear force on bending elements allows calculations to be made based on a common model.

This paper presents a design model for the strength of reinforced concrete slabs under punching, based on a unified approach to calculating the strength of reinforced concrete elements under shear force and punching.

Within the framework of the developed model, the maximum shear force acting on the slab is determined as the sum of the maximum shear forces perceived by two mutually perpendicular beams of variable width. It is assumed that the beams are loaded with concentrated forces at a distance from the conditional support equal to the horizontal projection of the inclined cracks that form the buckling pyramid. The criterion for beam failure is considered to be the displacement of concrete in the compressed zone above the critical inclined crack, and the internal forces in the inclined section are taken as the forces in the concrete of the tensile zone, the compressed zone at the continuation of the inclined crack, and the forces in the transverse reinforcement.

The obtained dependencies were used to determine the internal forces at the stage of failure during punching and the ultimate force perceived by the slab.

As part of the testing of the developed method, a comparison was made with the design of the slab's punching resistance using the method that formed the basis of Eurocode 2.

The concrete strength class and the ratio of the length of the load transfer plate to the height of the slab were taken as variable factors. The concrete strength varied in the range C20/25...C50/60, the ratio of the load transfer plate to the slab height varied in the range a_{sup}/d =0.7...3,0, and the reinforcement ratio varied in the range $\rho =$ 0.010 and 0.015.

The calculations showed that the developed method correctly reflects the effect on the strength of slabs when punching concrete strength and the relationship between the dimensions of the outer plate for load transfer and the height of the slab.

Keywords: punching shear; strength; design model; beams; slab; comparison

PROBLEM STATEMENT AND ANALYSIS OF PREVIOUS RESEARCH

The failure resulting from punching is the most dangerous type of failure of reinforced concrete slabs, as it is fragile in nature and occurs as a result of the continuous development of inclined cracks forming a punching pyramid (Fig. 1, 2).

Fig.1 Collapsed 4th floor slab at Pipers Car Park, Wolverhampton on 25/03/1997
Photo by Wood J. G.M [1]
Рис.1 Руйнування перекриття 4-го поверху на парковці у Вулверхэмптоні 25/1997
Автор фото Wood J. G.M [1]

a *b*

Fig. 2 Inclined cracks in a slab (a punching pyramid) after a punching shear failure.
Photo by: *a* - MacGregor, J.G [2]; *b* - Michael J. L. Egberts [3]
Рис. 2 Похилі тріщини в плиті (піраміда продавлювання) після руйнування при продавлюванні [2,3].
Автор фото: *a* - MacGregor, J.G [2]; *b* -Michael J. L. Egberts [3]

Numerous experimental studies have been devoted to the problem of the resistance of reinforced concrete slabs to punching, including tests of column-slab connections [3-16] and foundation slabs [15,16], including under long-term [9] and dynamic loading [10].

As a result of experimental studies, the main patterns of crack formation and failure of reinforced concrete slabs during punching have been established, as well as the influence of the main factors on strength - concrete strength, ongitudinal and transverse reinforcement [3,4,6,7,8,11,16,17], concrete types and aggregate sizes [12, 13].

Based on the experimental data obtained, design models and methods for calculating the strength of reinforced concrete slabs under punching shear were developed [18,19,20], including the use of the finite element method [21]. However, these and other methods for calculating strength, including those used in regulatory codes [22-25], are far from perfect, differ from each other [16, 26,27], and consider flat vertical sections rather than inclined sections, along which failure occurs (Fig. 2).

In view of the above, further development of methods for calculating the strength of reinforced concrete slabs under punching shear, which is the subject of this paper, appears to be a pressing task from both a scientific and practical point of view.

The object of research is the strength of reinforced concrete slabs under punching shear.

The purpose of the work is to develop a method for calculating the strength of reinforced concrete elements under punching based on the actual of crack formation, stress-strain state, and failure of elements, taking into account the influence of geometric dimensions, longitudinal and transverse reinforcement, and the strength characteristics of concrete and reinforcement.

MAIN MATERIAL AND RESULTS OF RESEARCH

Existing methods for calculating the strength of reinforced concrete slabs under punching, including those used in regulatory codes [22-25], do not reflect the actual nature of crack formation and failure along inclined cracks with the formation of a punching pyramid (Fig. 1) and are based on calculations using flat vertical sections rather than failure surfaces. The latter not always allows to evaluate bearing capacity of slabs adequately especially in cases when the dimensions and support and reinforcement conditions differ from conventional. At the same time, the similarity of character of reinforced concrete slabs destruction under punching shear action and beams under shear allows to make computations more precisely on the basis of general computational model.

Particularity of reinforced concrete beams resistance to action of shear is availability of several forms of destruction which take place while distance a from concentrated load point to the support is decreasing. For the relative distance of the concentrated load from the support $a/d > 1{,}5...2{,}0$ the failure happens as a result of concrete crushing in a compressed zone above a shear crack and, if $1{,}0...1{,}5 < a/d < 1{,}5...2{,}0$, as a result of the shear (diagonal tension) of concrete in a compressed zone above a shear crack and, if $1{,}0 < a/d$, as a result of concrete crushing in a inclined strut between the support and the concentrated load point. In light of above-stated, the most precision evaluation of computation of beams strength can be achieved for creation of design models applying to the concrete forms of failure.

In [28] the model is reviewed and the method of computation for case of failure as a result of crushing of concrete above a shear crack is developed. For failure of concrete as a result of crushing in a inclined strut between a support and a concentrated load point the known design model of deep beams and corbels can be used. As of case of failure of beams as a result of shear of concrete in a compressed zone above a shear crack up to now an inappropriate attention was given to creation of similar design models. At the same time, this form of beams failure is closest in character to failure of slabs under punching shear, that allows to conduct computations of beams and slabs because on the basis of a common approach.

In light of above-stated, the ultimate shear force V_{sl} perceived by a slab under punching shear can be considered as the sum of ultimate shear $V_{b,1}$ and $V_{b,2}$, received by two perpendicular beams of a variable wideness (Fig. 3):

$$V_{sl} = V_{b,1} + V_{b,2} \tag{1}$$

Here beams are considered as loaded by point concentrated force with distance to a conditional support equal to length of a sloping cut horizontal projection forming a pyramid of punching shear. Thus, the problem of shear computation of a slab is narrowed to calculation of ultimate shear received by corresponding beams for in time of failure as a result of concrete shear of a compressed zone above a shear crack.

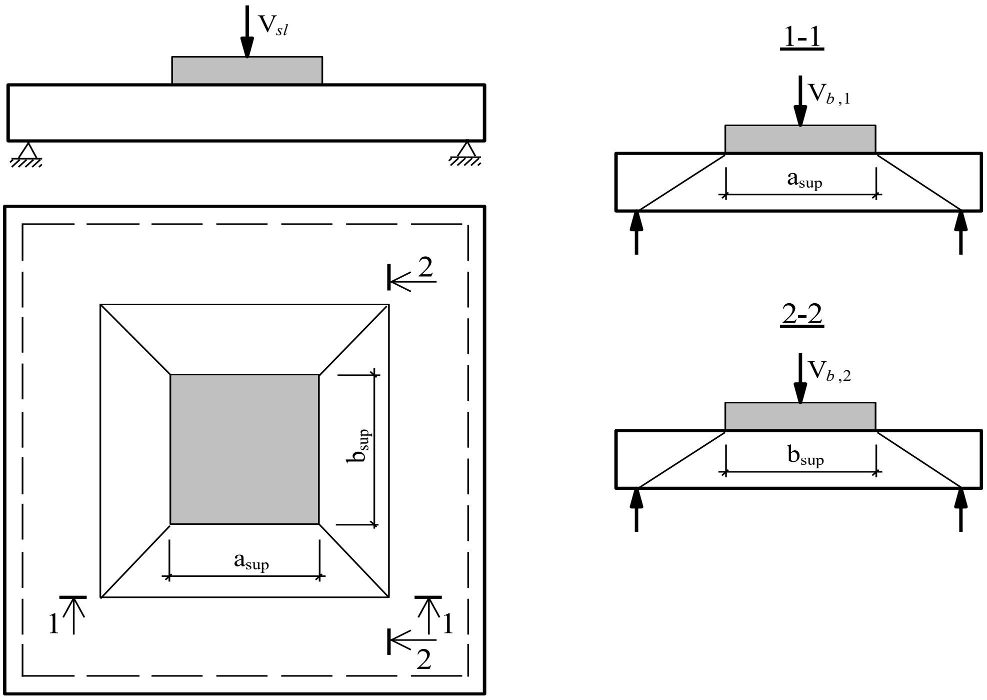


Fig.3 Representation of a slab in form of perpendicular beams
Рис.3 Представлення плити у вигляді перпендикулярних балок

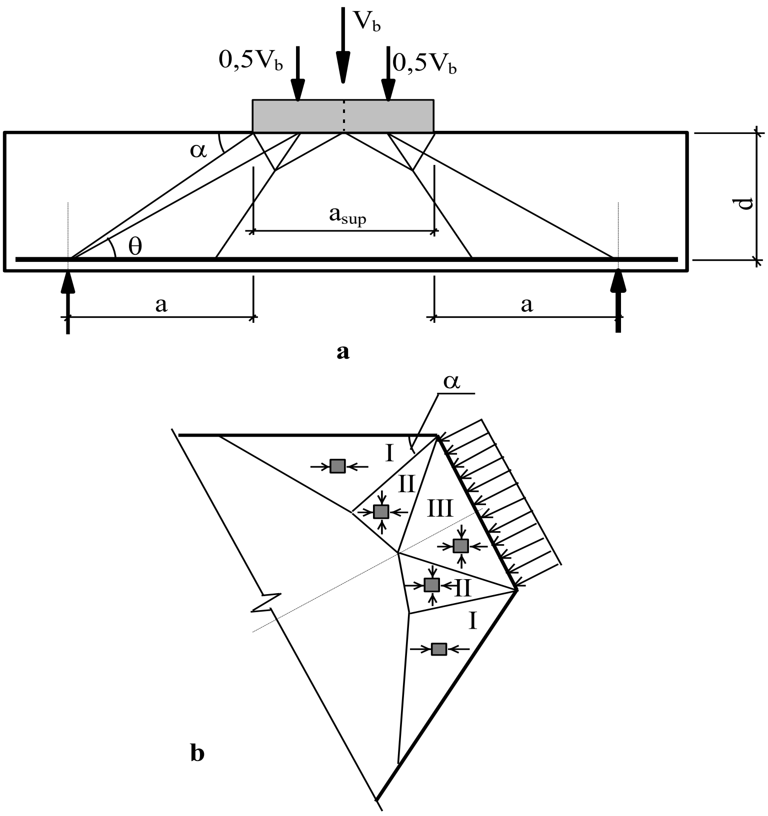
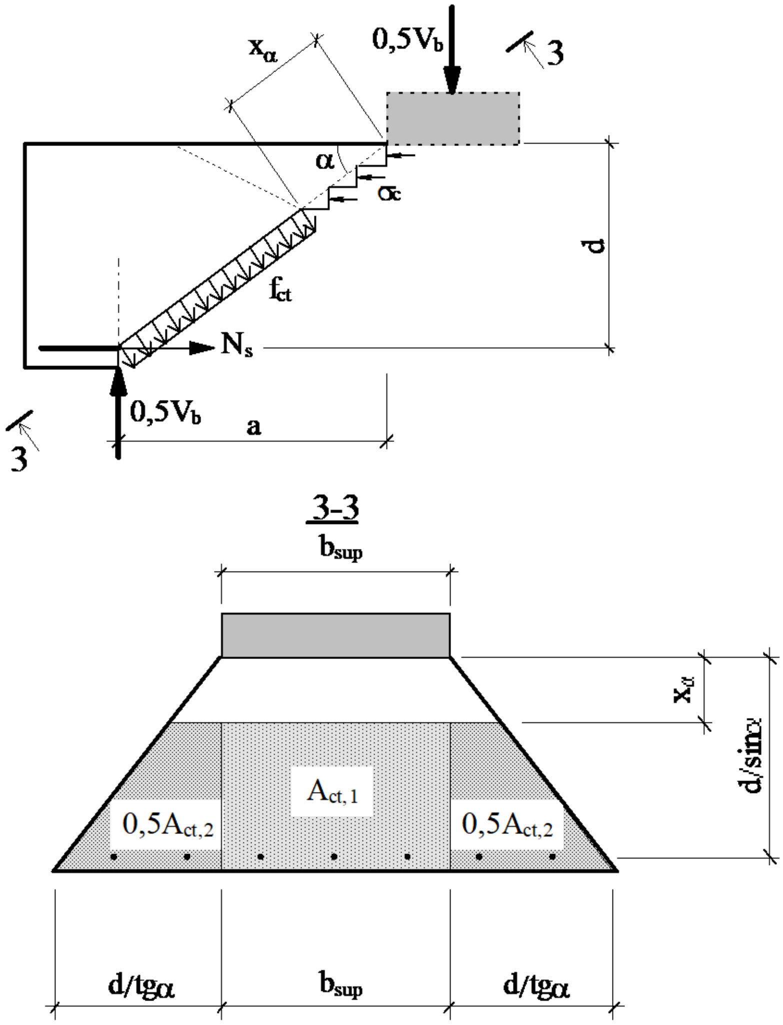


Fig.4 A beam design model (a) and strained state of concrete in compressed zone (b)
Рис.4 Розрахункова модель балки (а) та напружений стан бетону в зоні стиску (б)



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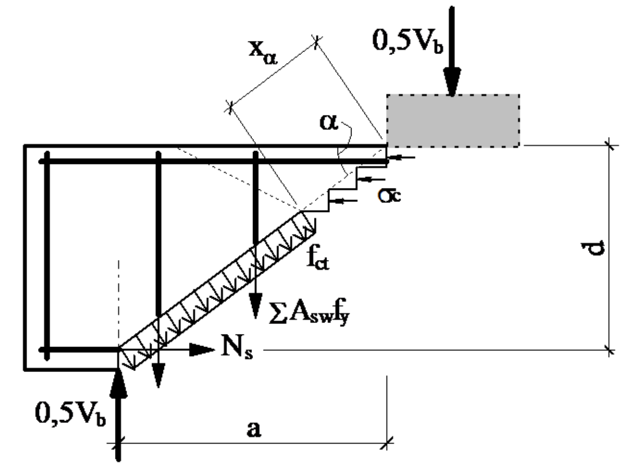

b

Fig. 5 -Design model of the beam ultimate limit state in a slopping section without punching shear reinforcement *(a)* and with punching shear reinforcement *(b)*

Рис.5 Розрахункова модель граничного стану балки в похилому перерізі без поперечного армування *(а)* і з поперечним армуванням на продавлювання *(б)*

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The accepted design model of a reinforced concrete beam during failure as a result of concrete shear of a compressed zone above a shear crack shown on Fig. 4 and Fig. 5. For possibility of a correct applying of the results obtained to computation of the slab under punching shear, the case where length of the transfer platform a_{sup} of the concentrated force is commensurable with height of a beam was considered.

The character of the concrete status in a compression zone was evaluated in view of, in terms of the theory of plasticity [29,30], the problem of action of an axial proportionally distributed load on a basis of the truncated concrete wedge. As a result, it is established, that the stressed status of a concrete wedge in a plane of loading is characterized by three areas (Fig. 4b): single axial compression in direction parallel to the wedge lateral faces (area I) and bi-axial irregular compression (areas II and III). Here particularities of the compressing for movement in direction from area I to areas II and III are increase.

Such the character of failure of compressing stresses allows to come to conclusion that the top of a shear crack is located on the area I boundary and the crack itself, as it is established as a result of numerous experiments, has a strait linear trajectory. The corresponding design model of a beam is represented in Fig. 5a, where as a criterion of the ultimate limit state the reaching values of tensile strength of concrete f_{ct} in a tensioned zone conterminous to the crack of a sloping section are accepted.

In light of the above-stated assumptions, the value of crack slope angle α and the height of concrete in a compressed zone above top of the crack (Fig. 5a) was accepted as for area I of a truncated concrete wedge (Fig. 4b).

Within the framework of an exact solution of the problem of the wedge angle α is determined the following formula:

$$\alpha = 0{,}5arccos(\frac{0{,}5 \cdot f_{cc} - f_{ct}}{1{,}5 \cdot f_{cc}}) \qquad (2)$$

For concrete of classes C12/15…C50/60 the angle α changes in the narrow limits: $36{,}9^0$ … $37{,}9^0$ and is close to experimental values of the inclination of the faces of slabs punching shear pyramid (Fig.2).

Height of a compressed zone of concrete above a shear crack x_α is at the rate of a wedge in function of sizes of its basis and angle of the edges. In the case considered, for length of the platform of the concentrated force transfer a_{sup} and slopping angle θ which is equal to (Fig. 5a):

$$\theta = arctg(ctg\alpha + 0{,}25 \cdot \frac{a_{sup}}{d}) \qquad (3)$$

Value x_α is about 0,75 of the wedge basis length and is calculated by (Fig. 5a):

$$x_\alpha = 0{,}75 \cdot 0{,}5 \cdot a_{sup} \cdot sin\theta = \\ = 0{,}375 \cdot a_{sup} \cdot sin\theta \qquad (4)$$

Force in a tensed zone of a sloping cut is calculated as square of a corresponding signal waveform of stretching stresses by:

$$N_{ct} = N_{ct,1} + N_{ct,2} = \\ = f_{ct} \cdot A_{ct,1} +\cdot f_{ct} \cdot A_{ct,2} \qquad (5)$$

Where:

$A_{ct,1}$- square of concrete of a tensed zone in the sloping cut of the beam of a variable wideness formed by plane of the pyramid of punching shear with the slopping angle within the limits of the concentrated force transfer platform wideness (Fig. 5b):

$$A_{ct,1} = (\frac{d}{sin\alpha} - x_\alpha) \cdot b_{sup} \qquad (6)$$

$A_{ct,2}$ - the sloping cut of the beam of a variable wideness formed by plane of the pyramid of punching shear with the slopping angle out of the limits of (Fig. 5b):

$$A_{ct,2} = (x_\alpha \cdot cos\alpha + \frac{d}{tg\alpha}) \cdot (\frac{d}{sin\alpha} - x_\alpha) \qquad (7)$$

ω - coefficient of the signal waveform in a sloping cut of the beam of a variable wideness out of the limits of the external load transfer platform breadth.

For calculation of the force N_{ct} by (5) the signal waveform of tensile stresses in a sloping cut of the beam was considered as an isosceles trapezoid with a rectangular site within the limits of the concentrated force transfer platform wideness and triangular sites outside of the site wideness. Such form of the signal waveform of stresses corresponds to the ω value equal 0,5.

The ultimate shear received by the beam is derived from an equilibrium equation of a sloping cut projected on a vertical axe (Fig.5a):

$$V_b = 2 \cdot N_{bt} \cdot cos\alpha \quad (8)$$

If the punching shear reinforcement is placed, it is taken into account within the limits of length of the sloping cut tensed zone horizontal projection (Fig. 5b). Here value of an ultimate shear is calculated by:

$$V_b = 2 \cdot (N_{bt} \cdot cos\alpha + \Sigma A_{sw} \cdot f_y) \quad (9)$$

where $\Sigma A_{sw} \cdot f_y$ - vertical force in punching shear reinforcement located within the limits of a sloping cut on length:

$$l_{sw} = (\frac{d}{sin\alpha} - x_\alpha) \cdot cos\alpha \quad (10)$$

To verify the accuracy of the developed method and the correctness of the influence of the main factors, a comparison was made with the calculations of the punching shear strength of unreinforced slabs in accordance with the method [30], which is based on Eurocode 2 [25].

The concrete strength class and the ratio of the length of the load transfer plate to the height of the slab were taken as variable factors. The concrete strength varied in the range C20/25...C50/60, the ratio of the load transfer plate to the slab height varied in the range a_{sup}/d =0.7...3,0, and the reinforcement ratio varied in the range $\rho = 0.010$ and 0.015.

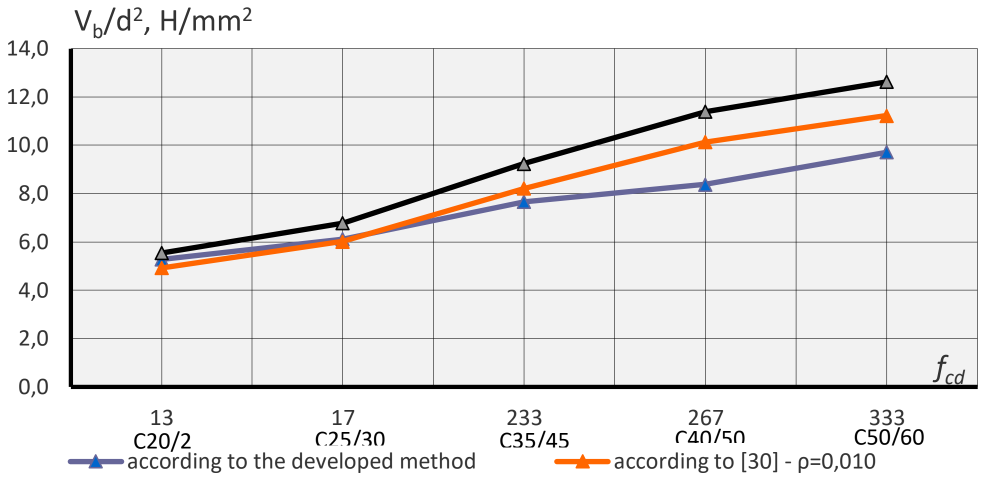
The results of the comparison as the schedules of relations of relative bearing capacity of the slabs and concrete strength and relative dimensions of the concentrated force transfer platform are indicated on Fig. 6.

The analysis of the results obtained has allowed to establish the following.

The results of the developed method are close to the calculations in [31] and correctly reflect the effect on the strength of slabs when the concrete strength is punched through (Fig. 6a, 6b) and the relationship between the dimensions of the external load transfer plate and the slab height (Fig. 6c). In this case, the design is based on [31] gives higher values strength of slabs, which is increased as the concentrated force transfer plate dimensions being increased (if $\rho = 0{,}010$) from 1…3 % if $a_{\sup}/d = 0{,}7$ up to 28 … 38 % if $a_{\sup}/d =$ 3,0. This may be due to the fact that [31] indirectly takes into account the increase in slab strength due to the dowel force in the longitudinal reinforcement by introducing a longitudinal reinforcement coefficient into the calculation. Within the framework of the developed method, the dowel action in the longitudinal reinforcement of slabs can be taken into account based on a general approach to determining transverse forces in the longitudinal reinforcement of reinforced concrete elements under shear [32,33].

CONCLUSIONS

The beams model of design of punching shear forces in reinforced concrete slabs on the basis the general approach to strength computation of shear appearing under punching shear is developed. Within the framework of the developed method the ultimate shear received by a slab under punching shear is determined as the sum of the ultimate shear received by two perpendicular beams of a variable wideness, see Fig.3. The ultimate shear force V_{sl} perceived by a slab under punching shear can be considered as the sum of ultimate shear V_{b1} and V_{b2}, received by two perpendicular beams of a variable wideness by (2).



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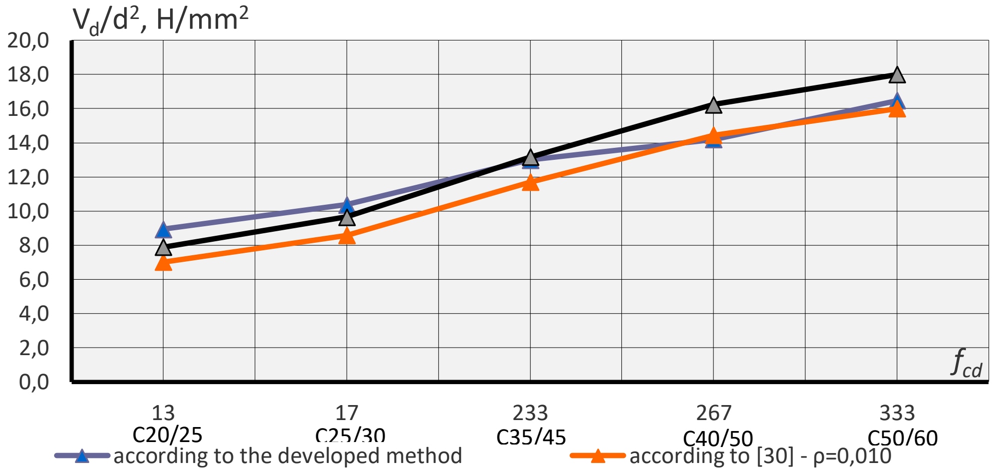

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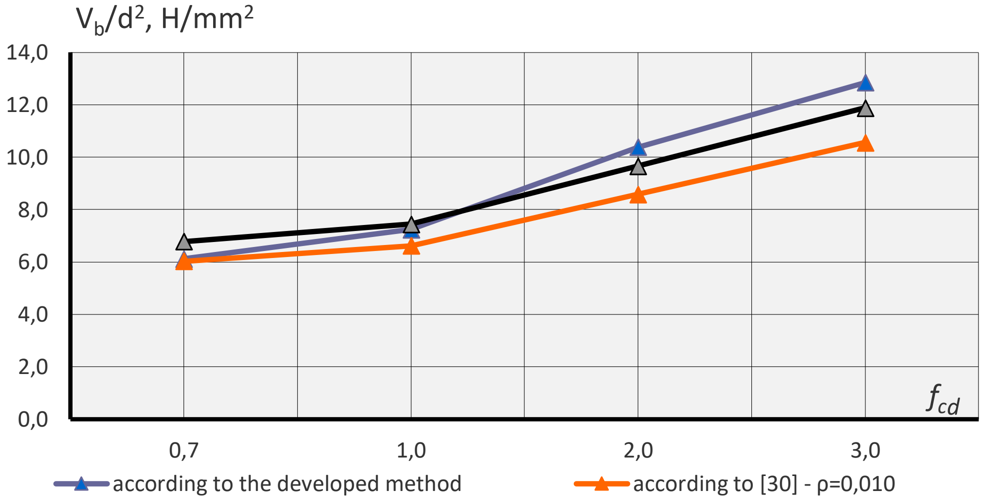

c

Fig.6 Dependence strength of slab under punching shear on concrete strength at $a_{sup}/d =$ 0,7 ***(a)*** and at $a_{sup}/d =$ 2,0 ***(b)*** and relative dimensions of the concentrated force transfer plate *(c)*.

Рис.6 Залежність міцності плити при продавлюванні від міцності бетону при $a_{sup}/d =$ 0,7 ***(а)*** і при $a_{sup}/d =$ 2,0 ***(б)*** і відносного розміру площадки передачі зосередженої сили ***(с)***.

The character of the concrete status in a compression zone was evaluated in view of, in terms of the theory of plasticity [29], the problem of action of an axial proportionally distributed load on a basis of the truncated concrete wedge, (Fig.4b). Criterion of the ultimate limit state the reaching values of tensile strength of concrete f_{ct} in a tensioned zone conterminous to the crack of a sloping section are accepted (Fig. 5b). Force in a tensed zone of a sloping cut is calculated as square of a corresponding signal waveform of stretching stresses by (5). The ultimate shear received by the beam is derived from an equilibrium equation of an a sloping cut projected on a vertical axe by (8).

The design value of the strength of slabs under punching shear according to the developed method is quite close to the results of the calculation according to [31], which is based on Eurocode 2 [25], but since the developed method is based on a general model of the ultimate state of elements under shear forces [28] and the theory of plasticity of reinforced concrete [29,30], unlike empirical relationships [31], it has broader application prospects.

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МІЦНІСТЬ ЗАЛІЗОБЕТОННИХ КОНСТРУКЦІЙ ПРИ ПРОДАВЛЮВАННІ

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Анотація. Продавлювання є одним із можливих видів руйнування залізобетонних конструкцій і виникає при прикладенні зосередженої сили до плит перекриття через площу, спів розмірну з висотою плити. Зокрема, коли колони спираються плити на перекриття, фундаментні плити та інші.

Існуючі методи розрахунку міцності залізобетонних плит на продавлювання базуються на емпіричному підході, який не завжди адекватно оцінює міцність плит і вплив основних факторів. Водночас загальний характер руйнування при продавлюванні та дії поперечних сил на згинальні елементи дозволяє проводити розрахунки на основі загальної моделі.

У цій статті представлено модель розрахунку міцності залізобетонних плит при продавлюванні засновану на загальному підході до розрахунку міцності залізобетонних елементів при дії поперечної сили та продавлюванні.

У рамках розробленої моделі максимальна поперечна сила, що діє на плиту, визначається як сума максимальних поперечних сил, що сприймають дві взаємно перпендикулярні балки змінної ширини. Вважається, що балки навантажені зосередженими силами на відстані від умовної опори, що дорівнює горизонтальній проекції похилих тріщин, які утворюють піраміду продавлювання. Критерієм руйнування балки вважається зсув бетону в стиснутій зоні над критичною похилою тріщиною, а у якості внутрішніх зусиль у похилому перерізі приймаються зусилля в бетоні розтягнутої зони, стиснутої зони на продовженні похилої тріщини і зусилля у поперечній арматурі.

В якості варійованих факторів були прийняті клас бетону за міцністю і відношення довжини площадки передачі зосередженої сили до висоти плити. Міцність бетону варіювалася в діапазоні C12/15…C40/50, відношення розмірів площадки до висоти плити приймалося в діапазоні a_{sup}/d =0.7...3,0, а коефіцієнт армування $\rho =$ 0,010 і $\rho =$=0,015.

В результаті розрахунків встановлено, що розроблений метод правильно відображає вплив на міцність плит при продавлюванні міцності бетону та співвідношення між розмірами зовнішньої пластини для передачі навантаження та висотою плити.

Ключові слова: продавлювання; міцність; розрахункова модель; балка; плита; співставлення

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