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COMPARISON OF THEORETICAL CALCULATED DEFLECTIONS ACCORDING TO THE EULER- BERNOULLI AND TYMOSHENKO BEAM MODELS WITH EXPERIMENTALLY OBTAINED

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Abstract. The article presents the results of an experimental and theoretical study of the prediction of deflections of timber beams made of three types of wood: massive, glued laminated and cross-laminated timber. The aim of the work is to compare the adequacy of the classical Euler-Bernoulli and the Timoshenko beam theory in predicting deflections under static loading.

Deflections of simply-supported beams under concentrated loading in the middle of the span were experimentally investigated. Experimental values of deflections and mechanical characteristics were determined for each type of timber. Theoretical deflections were calculated according to the Euler-Bernoulli and Tymoshenko beam theories for identical conditions.

Comparative analysis showed that the Euler-Bernoulli beam theory underestimates the deflections, with relative errors within 9%…15%, indicating a significant influence of shear deformations. On the other hand, the Timoshenko beam theory demonstrated much better convergence with experimental data, with errors within -2%…+4%.

To increase the accuracy of prediction by Euler-Bernoulli beam theory, it is proposed to introduce averaged empirical shear coefficients determined on the basis of experimental results. The application of these coefficients allowed to significantly reduce the discrepancies between the theoretical and experimental values of deflections for all studied timber types.

The obtained results confirm the importance of considering shear deformations in the analysis of beams made of timber-based materials. The application of the Timoshenko beam theory or the modified Euler-Bernoulli theory with considiration of empirical shear coefficients is more reasonable for accurately predicting their deformation behavior.

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Keywords: massive timber; glued laminated timber (glulam); cross-laminated timber (CLT); deformation modulus; shear modulus; Euler-Bernoulli beam theory; Timoshenko beam theory.

INTRODUCTION

In modern construction and reconstruction, significant attention is paid to the use of timber as an ecological and renewable construction material. Timber beams are important load-bearing elements in plenty types of structures, and accurate prediction of their behavior under load is critical to ensure the reliability and durability of structures. Despite the centuries-old history of the use of timber, the study of the peculiarities of its mechanical behavior and the application of modern theoretical models for its analysis remain relevant, taking into account the appearance in the last century of a number of new timber-based materials [1, 17].

In modern engineering analysis and design of structures, beams play a key role as elements that work in bending.

For adequate design and analysis of the behavior of beams under load, numerous theoretical models have been developed, each one has its own scope of application and is based on certain simplifying assumptions regarding the material, geometry and deformations of the element.

The Euler-Bernoulli beam theory (EBT) and the Timoshenko beam theory (TBT) stand out among the classical and most widely used theories. The Euler-Bernoulli theory, which is the foundation for many engineering calculations, is based on the hypothesis of the invariance of the normals to the neutral axis of the beam during deformation and the absence of shear deformations. Ehe Timoshenko beam theory takes into account the effect of shear deformation and rotational inertia, which makes it more accurate for the analysis of short and thick beams, as well as beams subjected to high-frequency dynamic loads [2, 3, 18, 19, 20].

Beside these fundamental models, there are other, more complex theories that include additional factors, such as the effects of curvature of cross-section, nonlinearity of the material, more complex cross-section shape. The development of computational technology has also led to the emergence of numerical methods, such as the finite element method (FEM), which allow to analyze beams with complex geometry and support/connection conditions without significant simplifying assumptions.The Rayleigh beam theory is an extension of the Euler-Bernoulli theory, which takes into account the rotational inertia of beam cross-sections [4, 5].

The Reddy-Bickford third-order beam theory is a variant of the higher-order beam theory that takes into account the effects of shear deformation during beam deformation. Unlike the classical Euler-Bernoulli beam theory, which neglects shear deformation, and the first-order Timoshenko beam theory, which assumes a constant distribution of shear strains over the cross-section, the Reddy-Bickford beam theory uses a cubic function to describe the longitudinal displacement along the thickness of the beam. This allows for the nonlinear distribution of shear strains and stresses to be considered, which is more realistic for thick beams or composite materials [15, 16].

High-order beam theory models extend Timoshenko theory by introducing additional degrees of freedom or higher derivatives for a more accurate description of deformations [6, 7].

Theories of composite beams - the analysis of such beams requires considiration of the features of their structure, such as the difference in the deformation modulus of the layers, the strength of the connection between the layers, etc. [8, 9].

Although the finite element method is not a “theoretical model” in the traditional sense (as a set of analytical equations), it is a powerful tool for the analysis of beams of any complexity, based on the discretization of a continuous system into a finite number of elements [10, 11].

In this article, as a continuation of research [12], the results of comparing experimental data and theoretical calculations according to the two most common beam theories: Euler-Bernoulli and Timoshenko are presented.

THE PURPOSE

To investigate the possibility of application of the Euler-Bernoulli and Timoshenko beam theories for theoretical calculations of deflections of beams made of three different types of timber: massive (MT), glued laminated (GLT), and cross-laminated timber (CLT).

To conduct a comparative analysis of experimentally obtained deflection values with the results of theoretical calculations according to both beam theories to assess their adequacy in predicting deformations of timber beams. Based on this comparison, to determine empirical refinement coefficients for the Euler-Bernoulli beam theory, which would consider the influence of shear deformation and allow to increase the accuracy of its predictions for the studied types of wooden beams.

THE MAIN CONTENT

Previously, three types of timber beams produced of local pine were experimentally investigated: massive, glulam and CLT (Fig. 1).

The experiments were performed for simply supported hinged beams with a concentrated load in the middle of the span. The calculated beam span $l = 1960$ mm, width $b = 90$ mm, height $h = 145$ mm.

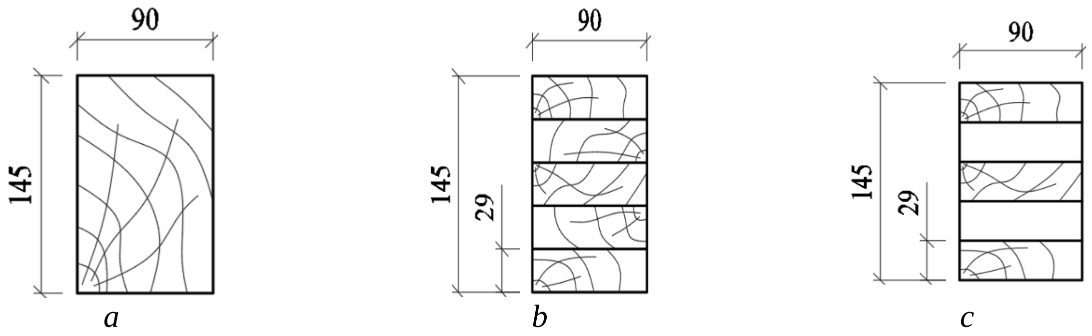

Fig. 1. Beams cross-section: *a* – massive; *b* – glulam; *c* – CLT
Рис.1. Перерізи балок: а - масивна; б - клеєна; в - CLT (клеєний багатошаровий конструктивний матеріал)

The deformation modulus, shear modulus and deflections are determined (Tables 1).

The Euler-Bernoulli beam theory is the most common and basic approach for analyzing the bending of long and thin beams.

Table 1. Experimentally determined deformation modulus and shear modulus
Табл.1. Модуль пружності та модуль зсуву

Timber type	E_{mean}, GPa	G_{mean}, GPa
MT	7.658	0.479
GLT	7.292	0.456
CLT	6.108	0.382

The main assumptions (Fig. 2):
- the beam is straight, prismatic and made of a linear-elastic homogeneous material;
- the cross-sections that were flat and perpendicular to the beam axis before loading remain flat and perpendicular to the beam axis after loading (shear deformation is ignored);
- the deformations are small;
- the loading acts in the bending plane;
- the beam axis is a neutral axis, where bending does not cause tension or compression.

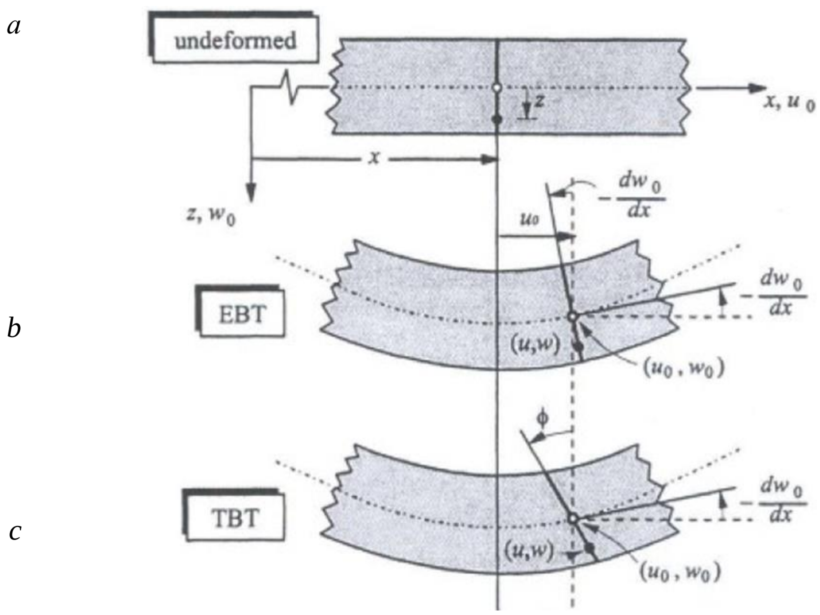

Fig. 2. Beams schemes: *a* – undeformed; *b* – Euler-Bernoulli beam; *c* – Timoshenko beam [13]
Рис.2. Схеми балок:а – недеформована;б – балка Ейлера-Бернуллі;в – балка Тимошенка [13]

The main differential equation of beam bending according to EBT [3]:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) - q(x) = 0 \quad (1)$$

where x – coordinate along the beam axis;
w – beam deflection;
EI – bending stiffness;
$q(x)$ – distributed load.

For a simply supported hinged beam subjected to a concentrated load applied at the midspan, according to the Euler-Bernoulli beam theory, the maximum deflection is determined by the expression [3]:

$$w_{max} = \frac{Pl^3}{48EI} \quad (2)$$

where P – concentrated load;
l – beam span.

The Timoshenko beam theory extends the classical Euler-Bernoulli theory by considering shear deformations and rotational inertia, making it more accurate in the analysis of short, thick, or high-frequency beams.

The main assumptions (Fig. 2):
- the beam is straight, prismatic and made of a linear-elastic homogeneous material;
- the cross-sections remain flat, but not necessarily perpendicular to the neutral axis of the beam after loading;
- small deformations and rotations;
- the rotational inertia of the cross-section considered;
- the loading acts perpendicular to the beam axis.

In the case of the Timoshenko beam theory, we have two main differential equations [3]:

$$\frac{d}{dx}\left[\kappa GA\left(\frac{dw}{dx} + \varphi\right)\right] + q(x) = 0 \quad (3)$$

$$\frac{d}{dx}\left(EI\frac{d\varphi}{dx}\right) - \kappa GA\left(\frac{dw}{dx} + \theta\right) = 0 \quad (4)$$

where x – coordinate along the beam axis;
w – beam deflection;
EI – bending stiffness;
$q(x)$ – distributed load;
φ – rotation of the beam section;
GA – shear stiffness;
κ – correction coefficient of shear stiffness (depends on the cross-section shape, $\kappa = 5/6$ for beams with rectangular cross-section).

The maximum deflection is determined by the expression [3]:

$$w_{max} = \frac{Pl^3}{48EI} + \frac{PL}{4\kappa GA} \quad (5)$$

Comparing the expressions for determining the deflections for EBT expression (2) and TBT expression (5), we see that in the expression of Timoshenko beams:
- the first part is the simple deflection for the Euler-Bernoulli beam, which takes into account only bending deformations;
- the second part is the deflection subjected to shear, which occurs due to the fact that the cross-section of the beam is deformed (this is ignored in the classical theory).

Виконавши порівняння виразів для визначення прогинів для ТБЕБ вираз (2) та ТБТ вираз (5), бачимо що у виразі балки Тимошенка:

For a more detailed analysis and the ability to trace the influence of the load level on the values of theoretical deflections determined by EBT and TBT, it was decided to perform calculations for three load levels: ~35%, ~70% and 100% of the maximum applied load during the experimental study. The calculation results are given in Table 2.

The theoretically calculated results were compared with the experimentally obtained ones. Having analyzed the theoretical deflection results, we can conclude that for the studied types of beams, the EBT gives a rather significant underestimation of deflections compared to the experimentally obtained ones: MT–10…12%; GLT–9…12%; CLT-14…15%.

On the other hand, calculated results by the TBT model, gives a fairly good correspondence between the theoretically calculated results and the experimental ones: MT – -0.6…1.3%; GLT – -1.8…0.8%; CLT – 3…4%.

Table 2. Experimentally determined δ_e and theoretically calculated deflections under load P_e according to the Euler-Bernoulli beam theories δ_{EB} and Timoshenko δ_T

Табл. 2. Експериментально визначені прогини δ_e та теоретично розраховані прогини під навантаженням P_e згідно з теорією балки Ейлера-Бернулліδ_{EB} та теорією балки Тимошенка δ_T

Timber type	P_e, kN	δ_e, mm	δ_{EB}, mm	$\frac{\delta_e - \delta_{EB}}{\delta_{EB}} \times 100\%$	δ_T, mm	$\frac{\delta_e - \delta_T}{\delta_T} \times 100\%$
MT	3.27	3.28	2.93	12.0	3.24	1.3
	6.10	6.00	5.46	9.8	6.04	-0.6
	8.81	8.74	7.89	10.7	8.72	0.2
GLT	3.27	3.34	3.08	8.6	3.40	-1.8
	6.10	6.28	5.74	9.4	6.34	-1.0
	8.81	9.23	8.29	11.4	9.16	0.8
CLT	2.62	3.37	2.94	14.5	3.25	3.6
	5.15	6.58	5.78	13.8	6.39	2.9
	7.11	9.14	7.99	14.4	8.82	3.6

For the possibility consideration of the effect of shear deformation for the Euler-Bernoulli beam theory, the principle described in [14] was applied, where by taking into account the shear deformation of the cross-section, lattice structures (trusses) were equalized to beam ones for the dynamic properties calculation. Consequently, the averaged shear coefficient k_G was applied. The coefficient k_G is determined by the ratio of the beam deflection determined by EBT δ_{EB} to the actual beam deflection under static loading δ_e.

$$k_G = \frac{\delta_{EB}}{\delta_e} \tag{6}$$

The determined coefficients k_G are performed in Table 3.

The averaged shear coefficient is aimed at reducing the stiffness characteristics of the beam according to the Euler-Bernoulli model and accordingly expression (2) takes the form:

$$w_{max} = \frac{Pl^3}{48EIk_{G,mean}} \tag{7}$$

where $k_{G,mean}$–mean value of the coefficients k_G determined for each type of beam

;

The k_G coefficients calculated according to expression (6) are given in Table 3.

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Table 3. Determined coefficients k_G
Табл. 3. Визначення коефыцыэнта k_G

Timber type	k_G	$k_{G,mean}$
MT	0.89	0.90
	0.91	
	0.90	
GLT	0.92	0.91
	0.91	
	0.90	
CLT	0.87	0.88
	0.88	
	0.87	

A comparison of the calculated deflections according to the refined Euler-Bernoulli beam theory, taking into account the shear deformation, by the averaged k_G coefficient with the experimentally obtained data was performed.

The comparison results are performed in Table 4.

Table 4. Experimentally determined δ_e and theoretically calculated deflections under load P_e according to the Euler-Bernoulli beam theory considering the shear coefficient $\delta_{EB,G}$
Табл. 4. Порівняння експериментально визначених прогинів δ_e з теоретично розрахованими за теорією Ейлера-Бернуллі з урахуванням зсувового коефіцієнта $\delta_{EB,G}$

Timber type	P_e, kN	δ_e, mm	$\delta_{EB,G}, mm$	$\frac{\delta_e - \delta_{EB,G}}{\delta_{EB,G}} \times 100\%$
MT	3.27	3.28	3.25	1.0
	6.10	6.00	6.06	-0.9
	8.81	8.74	8.75	-0.1
GLT	3.27	3.34	3.38	-1.1
	6.10	6.28	6.30	-0.3
	8.81	9.23	9.10	1.4
CLT	2.62	3.37	3.36	0.2
	5.15	6.58	6.61	-0.4
	7.11	9.14	9.12	0.2

The EBT model refined by applying the k_G coefficient showed a high correspondence of the theoretically calculated results with the experimental ones: MT–0…9-1%; GLT–1.1…1.4%; CLT – -0.4…0.2%.

CONCLUSION

The conducted experimental and theoretical study of the deflections of beams made of massive (MS), glued laminated (GLT) and cross-laminated timber (CLT) allowed us to compare the adequacy of the application of the classical Euler-Bernoulli beam theory (EBT) and the Timoshenko beam theory (TBT).

Analysis of theoretical calculations showed that the use of EBT leads to a significant underestimation of deflections compared to experimental data for all studied types of timber beams (MT – 10…12%; GLT – 9…12%; CLT – 14…15%). This indicates a significant influence of shear deformations on the actual deformation behavior of timber beams and composite beams made of timber-based materials.

On the other hand, the use of the Timoshenko beam theory demonstrated a high correspondence of the theoretically calculated deflections to the experimental values (MT – -0.6…1.3%; GLT–-1.8…0.8%; GLT –

3…4%). This confirms the accessibility of consideration of the shear deformations while analyzing beam elements made of timber-based materials to ensure high accuracy in predicting their behavior under load.

To increase the accuracy of predicting deflections according to the classical Euler-Bernoulli beam theory, an averaged empirical shear coefficient $k_{G,mean}$ was applied. The use of this coefficient allowed to significantly improve the convergence of theoretical and experimental results (MT – -0.9…1%; GLT – -1.1…1.4%; CLT – -0.4…0.2%). The determined averaged values of the shear coefficient $k_{G,mean}$ are: 0.90 for MT, 0.91 for GLT and 0.88 for CLT.

Thereby, the results of the study confirm that for accurate prediction of deflections of wooden beams, preference should be given to the theory of the Timoshenko beam or other theories that take into account the shear deformation. In cases where the Euler-Bernoulli theory is applied, it is recommended to use the proposed empirical shear coefficients to consider the influence of shear deformations and increase the accuracy of calculations.

For the further research, it will be advisable to investigate the influence of the cross-sectional dimensions, beam length, number and thickness of lamellas (for GLT and CLT) on the accuracy of calculations with the EBT and TBT models, as well as to determine and compare the shear coefficients $k_{G,mean}$.

Besides the stated above, the importance of this work is emphasized in the context of calculations of multilayer composite beams under dynamic and impulse loads. The obtained results simplify future analysis of experimental studies, in particular, due to the possibility of effective analysis of natural frequencies, which is crucial in modeling and predicting the dynamic behavior of structures.

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ПОРІВНЯННЯ РОЗРАХУНКОВИХ ПРОГИНІВ ПО МОДЕЛЯМ БАЛОК ЕЙЛЕРА-БЕРНУЛЛІ ТА ТИМОШЕНКА З ЕКСПРЕМЕНТАЛЬНО ОТРИМАНИМИ

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Анотація. У статті представлено результати експериментально-теоретичного дослідження прогнозування прогинів дерев'яних балок, виготовлених з трьох поширених типів деревини: масивної, клеєної та перехресно-клеєної. Метою роботи стало порівняння адекватності класичної теорії балки Ейлера-Бернуллі та теорії балки Тимошенка у прогнозуванні їхніх прогинів під статичним навантаженням.

Експериментально досліджено прогини шарнірно опертих балок при зосередженому навантаженні посередині прольоту. Для кожного типу деревини визначено експериментальні значення прогинів та механічні характеристики. Теоретично розраховано прогини за теорією балки Ейлера-Бернуллі та Тимошенка для ідентичних умов.

Порівняльний аналіз показав, що теорія балки Ейлера-Бернуллі недооцінює прогини, з відносними похибками до 9%…15%, що свідчить про значний вплив деформацій зсуву. Натомість, теорія балки Тимошенка продемонструвала значно кращу збіжність з експериментальними даними, з похибками в межах -2%…+4%.

Для підвищення точності прогнозування за теорією балки Ейлера-Бернуллі, запропоновано введення усереднених емпіричних коефіцієнтів зсуву, визначених на основі експериментальних результатів. Застосування цих коефіцієнтів дозволило суттєво зменшити розбіжності між теоретичними та експериментальними значеннями прогинів для всіх досліджуваних типів деревини.

Отримані результати підтверджують важливість урахування деформацій зсуву при аналізі балок з матеріалі. Застосування теорії Тимошенка або модифікованої теорії Ейлера-Бернуллі з емпіричними коефіцієнтами є більш обґрунтованим для точного прогнозування їхньої деформативної поведінки.

Ключові слова: масивна деревина (МД), клеєна деревина (КД), перехресно-клеєна деревина (ПКД), модуль деформації, модуль деформації зсуву, теорія балки Ейлера-Бернуллі, теорія балки Тимошенка

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